Presentation on

TEMPERATURE STOCHASTIC
MODELING FOR PRICING OF WEATHER
DERIVATIVE MARKET

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The main goal of this study is fourfold.

1. Weather derivatives are relatively young financial instruments that allow retail, insurance, construction, entertaining, agricultural and other types of companies to protect themselves from weather fluctuations.

2. These financial instruments, whose value and/or cash flows depend on the occurrence of some meteorological events, which are easily measurable, independently authenticable, and sufficiently transparent to act as triggering underlying for financial contracts.

3. This study is to construction of Temperature Stochastic Modeling for Pricing of Weather Derivative Market.
First, we begin our approach to briefly present the weather derivatives and weather derivatives market.

Next, we extend this weather derivatives market to construct the arbitrage – free prices of temperature future payoff.

Then, we construct a stochastic model for making bond with weather derivatives and financial derivatives (weather options).

Finally, we construct the mathematical model for market price of risk for weather derivatives.
A weather derivative is financial instruments, which are used to help a company or organization reduce the risks associated with adverse or unusual weather conditions.

They work just like most other derivative contracts except the underlying asset (which is a weather condition such as rainfall, temperature or snowfall) has no value with which to price the derivative contract.

Weather derivatives are used to hedge the risks of inclement weather conditions.

They are measurable and essentially triggered by actual weather conditions making them a predictable form of risk management.

Heating degree-day (HDD) and cooling degree-day (CDD) derivative contracts are among the most common types of weather derivatives.

The Chicago Mercantile Exchange (CME) provides a platform for buyers and sellers of weather derivatives and offer temperature, rainfall, snowfall, frost and hurricane derivatives contracts.

This paper is about the valuation of a certain class of financial contracts known as weather derivatives.

The purpose of weather derivatives is to allow businesses and other organisations to insure themselves against fluctuations in the weather.

Weather is not only an environmental issue but also a key economic factor, as recognized by the former US Commerce Secretary, Gary F. Locke, in 2011, when he stated that at least $485 BILLION of the world economy is weather sensitive.
1. WEATHER DERIVATIVE MARKET

The main temperature indices are so called Cooling Degree Days (CDDs), Heating Degree Days (HDDs) and Cumulative Average Temperature (CAT).

Weather derivatives are usually structured as swaps, futures, and options based on different underlying weather indices. Here, introduce some indexes frequently used on the weather derivatives market, which is the underlying of the temperature.

\[ T_i^{\text{max}} - T_i^{\text{min}} \]

The maximum and the minimum temperature (generally in degree Celsius) measured in one day \( i \).

\[ T_i = \frac{T_i^{\text{max}} + T_i^{\text{min}}}{2} \]

Mean Temperature of a day \( i \)

As above mentioned, one important underlying variables for weather derivatives is the degree – day. For a given location, the degree – day is the temperature value of the different between the temperature of given day and a temperature threshold.

\[ HDD_i = \max(T_{\text{ref}} - T_i, 0) \]

Measure of heat in Summer

\[ CDD_i = \max(T_i - T_{\text{ref}}, 0) \]

Measure of cold in winter
**Reference temperature** (in general between 18 Degree C and 20 Degree C predetermined temperature level.

**Average temperature** calculated from Max and Min Temperature on given day $i$.

We can write cumulative HDD (CHDD)

\[
CHDD = \sum_{i=1}^{N} HDD_i
\]

The CAT index is for main cities in a country. For a measurement period $[T_1, T_2], T_1 < T_2$, the CDD index is measuring the demand for cooling and is defined as the cumulative amount a threshold $c$.

\[
\sum_{t=T_1}^{T_2} \max[T_t - c, 0]
\]

- The threshold $c$ is in the market given as $18^\circ$C, or $65^\circ$F and is the trigger point for when air-conditioning is switched on.
- The measurement periods are set to weeks, months, or seasons consisting of two or more months, within the warmer parts of the year.
- A futures written on the CDD pays out an amount of money to the buyer proportional to the index, in US dollars for the American market, and in Euro for the European one.
An HDD index is defined similarly to the CDD as the cumulative amount of temperatures below a threshold $c$ over a measurement period $[T_1, T_2]$ that is,

$$\sum_{t=T_1}^{T_2} \max [c - T_i, 0]$$

The CAT index is simply the accumulated average temperature over the measurement period defined as

$$\sum_{t=T_1}^{T_2} T_i$$

For mathematical convenience, we will use integration rather than summation and define the three indices CDD, HDD, and CAT over a measurement period by $[T_1, T_2]$.

$$CDD(T_1, T_2) = \int_{T_1}^{T_2} \max [T_i - c, 0] dt$$

$$HDD(T_1, T_2) = \int_{T_1}^{T_2} \max [c - T_i, 0] dt$$

$$CAT(T_1, T_2) = \int_{T_1}^{T_2} T_i dt$$
# Links between Weather and Financial Risk

<table>
<thead>
<tr>
<th>Risk Holder</th>
<th>Weather Type</th>
<th>Risk</th>
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</thead>
<tbody>
<tr>
<td>Energy Industry</td>
<td>Temperature</td>
<td>Lower sales during warm winters or cool summers</td>
</tr>
<tr>
<td>Energy Consumers</td>
<td>Temperature</td>
<td>Higher heating/cooling costs during cold winters and hot summers</td>
</tr>
<tr>
<td>Beverage Producers</td>
<td>Temperature</td>
<td>Lower sales during cool summers</td>
</tr>
<tr>
<td>Building Material Companies</td>
<td>Temperature/Snowfall</td>
<td>Lower sales during severe winters (construction sites shut down)</td>
</tr>
<tr>
<td>Construction Companies</td>
<td>Temperature/Snowfall</td>
<td>Delays in meeting schedules during periods of poor weather</td>
</tr>
<tr>
<td>Ski Resorts</td>
<td>Snowfall</td>
<td>Lower revenue during winters with below-average snowfall</td>
</tr>
<tr>
<td>Agricultural Industry</td>
<td>Temperature/Snowfall</td>
<td>Significant crop losses due to extreme temperatures or rainfall</td>
</tr>
<tr>
<td>Municipal Governments</td>
<td>Snowfall</td>
<td>Higher snow removal costs during winters with above-average snowfall</td>
</tr>
<tr>
<td>Road Salt Companies</td>
<td>Snowfall</td>
<td>Lower revenues during low snowfall winters</td>
</tr>
<tr>
<td>Hydro-electric power generation</td>
<td>Precipitation</td>
<td>Lower revenue during periods of drought</td>
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</table>
3. ARBITRAGE – FREE PRICING OF DERIVATIVES

Future on traditional assets such as stocks, bonds, agricultural and most energy products are priced under the cost of carry approach.

The logic of this approach is that there are two alternative for obtaining the asset in question at some point in the future.

These are either, borrow to purchase it now and store asset, or agree to purchase the asset at that later date via future contract, under the absence of arbitrage, the cost of both approaches should be equivalent.

Hence, the current cost of a futures contract is related to the current price of the asset and cost of borrowing and storing the asset.
In the CME market, we have temperature futures written on CAT, HDD and CDD indices introduced in section 2. We rewrite the temperature indices in a continuous way for the convenience for pricing

\[
\begin{align*}
CAT(\tau_1, \tau_2) &= \int_{\tau_1}^{\tau_2} T_i dt \\
CDD(\tau_1, \tau_2) &= \int_{\tau_1}^{\tau_2} \max[T_i - c]^+ dt \\
HDD(\tau_1, \tau_2) &= \int_{\tau_1}^{\tau_2} \max[c - T_i]^+ dt
\end{align*}
\]

Derivative for a measurement period for the year \( n+1 \), we would calculate the fictive indices the same derivative had in the year \( n, n-1, n-2 \) etc... and \( c \) is the reference temperature for CDD and HDD indices, and \( T_i \) is the daily average temperature DAT on time \( t \).

\( T_i \) is modelled as defined Siting Huang [2014] by a Levy driven Continuous – time Autoregressive (CAR) process with seasonal volatility where \( T_i \) satisfies

\[
T(s) = \Lambda(s) + e_1^T A(s-t) X(t) + \int_t^s e_1^T A(s-u) e_p \sigma(u) dL(u)
\]

\( s \geq t \geq 0 \)
• Temperature market is a classical example of incomplete markets since the underlying temperature indices for temperature futures are not tradable.
• Moreover, we consider Levy processes in our model and this introduces a higher level of incompleteness in our market model.

Arbitrage Theory
• Here we rely on the arbitrage theory to study the price of temperature derivatives.
• Under the no-arbitrage assumption, the price processes of all tradable assets in the temperature market should be arbitrage-free.

Equivalent Martingale Measure
• An equivalent martingale measure (EMM) $Q$ exists, which is a probability measure equivalent to $P$ such that the discounted price processes of all tradable assets are martingales with respect to $Q$.
• Because the market is incomplete, there are infinitely many EMMs.

Temperature Futures
• For the market of temperature futures, the risk-free bond with interest rate $r$ is the only tradable asset.
• Using the bond price as a numeraire, the discounted bond price is 1 and it is a trivial martingale with respect to all equivalent measures. Thus, all equivalent measures $Q$ are EMMs.
The temperature futures are traded in the market and their price processes should be arbitrage-free. The buyer of the futures enters the contract at time \( t \) and will receive a random amount \( Y \) in return of futures price.

\[
0 = e^{-r(\tau_2 - t)}E_Q[Y - F(t, \tau_1, \tau_2)|F_t]
\]

For each time \( t \), where \( Q \) is an EMM measure. The future price is then given by

\[
F(t, \tau_1, \tau_2) = E_Q[Y|F_t]
\]

Therefore, the price of the CAT, CDD and HDD futures at time \( t \) are, respectively,

\[
F_{\text{CAT}}(t, \tau_1, \tau_2) = E_Q\left[\int_{\tau_1}^{\tau_2} T(u) \, du | F_t\right]
\]

\[
F_{\text{CDD}}(t, \tau_1, \tau_2) = E_Q\left[\int_{\tau_1}^{\tau_2} (T(u) - c)^+ \, du | F_t\right]
\]
4. WEATHER OPTION CONTRACTS

The most common of these contracts come in the form of either Heating Degree Days (HDDs) or Cooling Degree Days (CDDs) contracts.

The payoff of these contracts is based on the cumulative in the daily temperature relative to 18°C (about 64°F) over a fixed period such as a month.

The fixed level of 18°C is the temperature at which the energy sector believes little heating or cooling occurs in households.

Outside the CME, there are a number of different contracts traded on the OTC market. One common type of contract is the option. There are two types of options, calls and puts.

The buyer from a positive payoff if cumulative temperature is below or above a specified level.

Often the option has a cap on the maximum payoff unlike, for example, traditional options on stocks.

The buyer of a HDD call, for example, pays the seller a premium at the beginning of the contract.

In return, if the number of HDDs for the contract period is greater than the predetermined strike level the buyer will receive a payoff.

The tick size is the amount of money that the holder of the call receives for each degree-day above the strike level for the period.
A generic weather option can be formulated by specifying the following parameters:

- **Strike Level**
- **Tick Size**
- **Contract Period**

The formula for the payoff of an option, then the number HDDs and CDDs for the period:

\[ H_n = \sum_{i=1}^{n} HDD_i \quad \text{and} \quad C_n = \sum_{i=1}^{n} CDD_i \]

Now we can write the payoff of an uncapped HDD call as:

\[ \chi = \alpha \max\{H_n - K, 0\} \]
Nature quantitative indices derived from daily temperature measurements, which are designed to reflect the demand for energy needed to heat or cool houses and factories.

The consists in the fact that heating is usually required when temperature drops below some reference level and thus energy expenditure is needed.

Requirements for a given subject at a specific geographical location are commonly considered directly proportional to the number of degree-days.

HDD is defined as the number of degrees by which the daily average temperature is below some base temperature, while CDD express the number of degrees by which the daily average temperature is above this value.

Mathematically expressed, daily HDD and CDD structures look as follows:

\[
HDD = \max \left( 0, T_{\text{base}} - T_{\text{Daily-Average}} \right) \quad \forall T(t) < 65
\]

\[
CDD = \max \left( 0, T_{\text{Daily-Average}} - T_{\text{base}} \right) \quad \forall T(t) > 65
\]

Considering the interest of natural gas companies, increase in HDD corresponds with decreasing temperature, which is reflected in higher natural gas consumption.

Since weather derivatives are applied for hedging of weather related risks over a long horizon, production of aggregate indices is needed.

The importance of a cumulative HDD index (Next Equation), which includes both nonlinear transformation of daily average temperature into HDD as well as further aggregation of daily indices because:
Weather derivatives are typically written on a cumulative sum of weather related outcomes.

November-March HDD contract is one of the most actively traded weather-related contracts and is also of a substantial interest to end users of weather models.

\[
\text{Cumulative HDD} = \sum_{t=1}^{n} \max(T_{\text{base}} - T_{t}, 0)
\]

- **Define** how weather variability is encapsulated for the purposes of a weather derivative contract.
- The contract is then financially settled using the measured value of the index as the input to a pay-off function.

**Contract**

- This function defines precisely who should pay what to whom at the end of the contract.
- Any function could be used as a pay-off function, but in practice only a small number of simple structures, with straightforward economic purposes, are common.

**Payoff**

- We will consider the payoff for each of these structures from the point of view of the buyer of the contract, who is said to take the ‘long’ position.
- The seller of the contract, who takes the ‘short’ position, will have exactly the opposite pay-off.
As we have already stated, the most commonly used weather derivatives are options, especially calls and puts as well as their various combinations, e.g. collars. With regard to intentions of particular hedges, a company generally decides on various option types shown in Table 2. Beside various purposes of particular options, this table introduces also simple drafts of payoffs that are generally based on the difference between the exercise and actual level of a weather index.

<table>
<thead>
<tr>
<th>Option type</th>
<th>Protection against</th>
<th>Exercise when</th>
<th>Payout</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDD Call</td>
<td>Overly Cold Winters</td>
<td>HDD &gt; Strike</td>
<td>Tick*(HDD-Strike)</td>
</tr>
<tr>
<td>HDD Put</td>
<td>Overly Warm Winters</td>
<td>HDD &lt; Strike</td>
<td>Tick*(Strike-HDD)</td>
</tr>
<tr>
<td>CDD Call</td>
<td>Overly Hot Summers</td>
<td>CDD &gt; Strike</td>
<td>Tick*(CDD-Strike)</td>
</tr>
<tr>
<td>CDD Put</td>
<td>Overly Cold Summers</td>
<td>CDD &lt; Strike</td>
<td>Tick*(Strike-CDD)</td>
</tr>
</tbody>
</table>

**Table 2 – Temperature Options**
As we are primarily interested in fluctuations of natural gas consumption during winters, CDD are not considered in this paper as this index corresponds especially to requirements for cooling energy.

In spite of wide flexibility available in designing weather derivatives, basic attributes are common for the majority of contracts.

Therefore, several basic features have to be specified to determine the payoff from a HDD option.

On the day following the end of a contract period, the payoff from an option may be computed in compliance with following equations.

For Call option

\[ V = \min\left(\max\{0, (HDD - X)\} \times \text{tick, cap}\right) \]

For Put option

\[ V = \min\left(\max\{0, (X - HDD)\} \times \text{tick, cap}\right) \]
Where \( X \) means the strike level, HDD is an aggregate level of the index, Tick is the payment per one HDD and Cap is the maximum payment from an option.

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cincinnati</td>
<td>Ohio</td>
<td>1/5/09</td>
<td>31/10/09</td>
<td>CDD Put</td>
<td>985</td>
<td>15,000</td>
<td>3 Mil</td>
<td>335,000</td>
</tr>
</tbody>
</table>

* Measure in CDD, determine the exercising of the option

Figure 3: Exemplary payoff diagram for a Long Put
5. MARKET RISK PREMIUM

Here we construct the mathematical model for the market risk premium and how it can be further explored to study the prices behaviour in the weather derivative market.

The temperature derivatives market is a classical incomplete market, and there are infinite many EMMs.

Before we have introduced a parametric class of measures by Esscher transform, where parameter is square integrable functions in our assumption.

The price of the derivatives depends on the selected EMM, and hence on the parameter \( \theta(t) \).

\( \theta(t) \) is interpreted as the market price of risk (MPR). The choice of reflects the preference towards risk of the market participants.

Recent studies have found that the MPR is different from zero.

In order to study the significance of the MPR, we can calibrate the MPR from market data.

The calibration process is in fact the inverse problem to the pricing process.

The estimated value of \( \theta \) gives the theoretical prices that are closest to the market prices according to some minimizing criterion.
Assume we have different futures contracts

\[ i = 1, \ldots, I \]

With measurement period \( [\tau_1, \tau_2] \)

For a single trading date \( t \), or the so called close date, we observe market prices

\[ F^*(t, \tau_1^i, \tau_2^i) \]

\[ i = 1, \ldots, I \]

We restrict to the case where the MPR is constant for each contract per trading date, i.e.

The aim to minimize the objective function given as the squared distance between,

The observed market price

\[ F(t, \tau_1^i, \tau_2^i) \]

Theoretical price

\[ F^*(t, \tau_1^i, \tau_2^i) \]

Estimated under the pricing framework, i.e. we solve the problem

\[
\text{Min} \left\{ F(t, \tau_1^i, \tau_2^i) - F^*(t, \tau_1^i, \tau_2^i) \right\}^2
\]
Since, from the closed form pricing formula for CAT futures under the Levy driven CAR model, we have that:

\[
\theta_{\text{CAT}}(t) = \arg \min_{\theta(t)} \left\{ \int_{\tau_1}^{\tau_2} \sigma(u) \mathbf{a}(t, \tau_1, \tau_2) \mathbf{e}_p du + \right. \\
\left. \left( \int_{\tau_1}^{\tau_2} \sigma(u) \mathbf{e}_1^T \mathbf{A}^{-1} \left( e^{\mathbf{A}(\tau_2-u)} - I_p \mathbf{e}_p \right) du \right) \right\}
\]

Hence we obtain the MPR \( \theta(t) \) which is varying with respect to time \( t \) but it is constant on \( [t, \tau_2^i] \). This framework can be generalized with more complex formulations on \( \theta(t) \).

- For example, Hardle and Cabrera [2012] consider formulations of **one piece constant, two piece constant**, and a general form of series expansion for **under the Brownian driven CAR model**.
- We can put similar considerations here for **CAT futures**, since it also has an explicit expression with cumulate function of the **Levy process**.
- It is worth mentioning that one usually calibrates the MPR for temperature futures and use the implied MPR for further pricing of options.

In fact, the market price of risk is associated with the economic concept of risk premium, defined as the difference between the risk neutral price

\[
RP = F(t, \tau_1, \tau_2) - F_p(t, \tau_1, \tau_2)
\]

which is the expected pay-off under \( Q \), and the price

\[
F_p(t, \tau_1, \tau_2)
\]

which the expected payoff under physical measure \( P \), i.e.,
CONCLUSION

Weather derivatives are an interesting extension of the derivative market.

In contrast to traditional derivatives, the underlying of weather derivatives, temperature, precipitation, wind, etc., cannot be traded solely.

Since weather variables are mostly uncorrelated with the classical financial market, weather derivatives form the only possibility on the financial market to insure against unfavourable weather.

The development of the weather derivatives market supposes that an increasing number of corporations take advantage of these new opportunities.

We described an approach to briefly present the weather derivatives and weather derivatives market, and then we extended this weather derivatives market to construct the arbitrage – free prices of temperature future payoff.

We constructed the temperature stochastic modelling for pricing of weather derivatives market.
THANK YOU

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