Presentation on

HOW SHOULD WE USE THE CONCEPT OF ENTROPY IN FINANCIAL MARKETS – PHYNANCE APPROACH

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AGENDA

1. Current Research
2. Related Study (Field)
3. Economics and Ideal Gas (Basic relation B/W Thermodynamics and Finance/Economics)
4. Thermodynamics Process and Their Economic Interpretation
5. The Main Goal of this Study

Relation between the Finance and physics. (PHYNANCE)

Concept of entropy in Economic systems

Here we are exploring the interpretation of entropy in finance.

Then we extend the concept of entropy used in finance with standard economic utility theory by using of entropy and its maximization

To construct the model of money entropy and interest rates.

And this paper ends with a conclusion
CURRENT RESEARCH


RELATED STUDY (FIELD)

The application regarding the various aspects about **Econophysics**, **Thermoeconomics**, **Ecodynamics** and **Phynance** within economics is very essential.

<table>
<thead>
<tr>
<th><strong>Econophysics</strong></th>
<th>• Within financial economics which attempts to express the experienced relationships within physics and economics.</th>
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</thead>
<tbody>
<tr>
<td><strong>Thermoeconomics</strong></td>
<td>• Biophysical economics, a heterodox economics which makes use of thermodynamics laws to economic theory.</td>
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<tr>
<td><strong>Ecodynamics</strong></td>
<td>• Components within applied economics. With regard to various sources, ecodynamics covers knowledge regarding monetary value, money flow and money usage.</td>
</tr>
<tr>
<td><strong>Phynance (Physics Together with Finance)</strong></td>
<td>• Field which incorporates both finance and physics aspects for the purpose of facilitating proper understanding promotion concerning various economic aspects.</td>
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<tr>
<td><strong>Brownian motion</strong></td>
<td>• (Upon Capital Market) may be perceived as stuff bits which are usually kicked around through some atoms’ forces.</td>
</tr>
<tr>
<td><strong>Quantum Finance</strong></td>
<td>• Is an interdisciplinary research field, applying theories and methods developed by <strong>quantum physicists and economists</strong> in order to solve problems in finance.</td>
</tr>
<tr>
<td><strong>Thermoney</strong></td>
<td>• Application of thermodynamics principle in <strong>money (Monetary Economics)</strong>.</td>
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</table>
## ECONOMICS AND THE IDEAL GAS

<table>
<thead>
<tr>
<th>Thermodynamics</th>
<th>Economics</th>
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</thead>
<tbody>
<tr>
<td>The Ideal Gas Equation</td>
<td>The Ideal Economic Equation</td>
</tr>
<tr>
<td>( PV = NkT )</td>
<td>( PV_t = NkT_t )</td>
</tr>
<tr>
<td><strong>P</strong> - Pressure on the wall of the system</td>
<td><strong>P</strong> – Price at the Boundary of the system (Unit Price)</td>
</tr>
<tr>
<td><strong>V</strong> – Contained in Volume</td>
<td>( V_t ) - Volume flow (Over a period of Time - Output Volume)</td>
</tr>
<tr>
<td><strong>N</strong> – Number of Molecules in a Ideal Gas System (Amount of Substance)</td>
<td><strong>N</strong> – Number of Shares in Issue (Number of Monetary Units)</td>
</tr>
<tr>
<td><strong>k</strong> – Boltzmann Constant</td>
<td><strong>k</strong> – Fixed for the investors (Constant) – Defining the Nominal value of a Monetary Unit (1$, 1 €)</td>
</tr>
<tr>
<td><strong>T</strong> – Temperature of the Ideal Gas</td>
<td><strong>T_t</strong> – Index (or a Degree of a Scale) of Trading Value over a period of time</td>
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</table>
Thermodynamic cycle consists of such thermodynamic processes that enable the system to return to its initial state, so it can be presented as a closed curve.

The analysis of heat engines in which gas is the working substance demonstrates that the thermodynamic cycle can be divided into the process of decompression or expansion of a gas and the process of compression or contraction of a gas.

Work performed by the gas in the thermodynamic power cycle and its economic interpretation. Thermodynamic and economic processes in a theoretical approach.
<table>
<thead>
<tr>
<th>Processes</th>
<th>Thermodynamic interpretation</th>
<th>Economic interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PV = \text{constans}$</td>
<td>isothermal process – absolute work done during the process is equal to the amount of heat absorbed by the gas ($dW = dQ$), it is the most efficient process, however it is difficult to be achieved</td>
<td>process during which the total amount of expenditures or turnovers (or the circulation velocity of money) is constant – it resembles movement along a unit elastic curve of demand</td>
</tr>
<tr>
<td></td>
<td>$(\frac{P_2}{P_1} = \frac{V_1}{V_2})$</td>
<td></td>
</tr>
<tr>
<td>$V = \text{constans}$</td>
<td>isochoric process – absolute work done during the process is equal to zero ($dW = 0$)</td>
<td>process during which the total output volume is constant – it resembles movement along a perfectly inelastic curve of supply (or demand)</td>
</tr>
<tr>
<td></td>
<td>$(\frac{P_2}{P_1} = \frac{T_2}{T_1})$</td>
<td></td>
</tr>
<tr>
<td>$P = \text{constans}$</td>
<td>isobaric process – absolute work done during the process is proportional to the increase of volume ($dW = P(V_2 - V_1)$)</td>
<td>process during which the price level is constant – it resembles movement along a perfectly elastic curve of supply (or demand)</td>
</tr>
<tr>
<td></td>
<td>$(\frac{V_2}{V_1} = \frac{T_2}{T_1})$</td>
<td></td>
</tr>
<tr>
<td>$PV^m = \text{constans}$</td>
<td>polytropic process* – absolute work done during the process is equal to the decrease of the internal energy of the gas ($dW = -dU$),</td>
<td>process during which the total amount of expenditures or turnovers (or the circulation velocity of money) is changing proportionately – it resembles movement along an isoelastic curve of demand</td>
</tr>
<tr>
<td></td>
<td>$(\frac{P_1}{P_2} = (\frac{V_2}{V_1})^m)$</td>
<td></td>
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</table>

Characteristic (reversible) thermodynamic processes of ideal gases in closed systems and their economic interpretation.
THE MAIN GOAL OF THIS STUDY IS FIVE FOLD:
The overriding objective of this research paper is to carry the convergence/unification of physics and finance program, i.e. Phynance.

In this work, we attempt can the concept of entropy in thermodynamics be applied to financial markets.

1. Begin our approach through the relation between the finance and physics.

2. Next here we introduce the concept of entropy in Economic systems

3. Here we are exploring the interpretation of entropy in finance

4. Then we extend the concept of entropy used in finance with standard economic utility theory by using of entropy and its maximization.

5. Finally, we construct the model of money entropy and interest rates.

And this paper ends with a conclusion.
INTRODUCTION (Link BW Thermodynamics and Finance/Economics)

The specialty of “physics” is the study of interactions between the various terminologies of matter and its elements. The development of this subject over the last several centuries has led to a gradual refining of our understanding of natural phenomena.

In physics, financial markets are described as complex dynamics systems governed by many interacting elements.

Though at a nascent stage, the winds of convergence of physics and finance are unmistakably perceptible with

Several concepts of fundamental physics like quantum mechanics, field theory and related tools of non-commutative probability, gauge theory, path integral etc.

For pricing of contemporary financial products and for explaining various phenomena of financial markets like stock price patterns, critical crashes etc.
The cardinal contribution of physicists to the world of finance came with the Nobel Prize for economics in 1997 together with Robert Merton. The standard route to pricing of derivatives and similar financial assets is through the stochastic calculus and Ito’s Lemma that leads to the celebrated Black Scholes formula for option pricing.

Option Pricing Formula

Pricing of financial derivatives by converting the problem into a heat equation and then solving it for specific boundary conditions.
A comprehensive theory of quantum mechanics has also been developed as a theory of ‘random walks’ (Brownian Motion). – Quantum Finance

The contemporary candidate for a unified theory of the four fundamental forces of Nature

- Electromagnetic, Gravitational, Electroweak and Strong interaction (i.e. string theory) also makes extensive use of random surfaces.

- Significant work is also focused on the convergence of the physical science with finance and economics.

- The inference of the possible existence of an underlying symmetry between financial markets and the fundamental theories of physics.

- This opens the way to using some of the well developed physical and geometrical methods in the analysis of financial markets.

- Attempts have also been made to develop the dynamics of financial markets in the Lagrangian and Hamiltonian formalism.

- Sparse work has also been done in applying the maxims of quantization, to the economics of financial markets.
The laws of thermodynamics can be considered as sayings of a mathematical model, and the fact that they are based upon commonplace observations makes them extremely powerful and generally valid.

In particular, the interest of applying thermodynamics in a systematic manner to describe the behavior of economic and financial systems has a long history.

<table>
<thead>
<tr>
<th>Thermodynamic</th>
<th>Economic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entropy, Temperature, Pressure and volume</td>
<td>Production, Effective costs, Capital growth, and Capital.</td>
</tr>
<tr>
<td>Statistical theory for large atomic systems under constraints of energy.</td>
<td>Large system of economic agents and goods under the constraints of capital.</td>
</tr>
</tbody>
</table>

In a recent review, the concepts of thermodynamics are shown to be applicable to economic systems and a consistent thermodynamic formulation of economics is presented, using the language of calculus and differential forms and the fundamentals of statistics.
Both systems may be handled by the Lagrange principle, the law of statistics for large systems under constraints. **Thermodynamics and economics** are expected to follow the same concept:

<table>
<thead>
<tr>
<th>Laws</th>
<th>Thermodynamics</th>
<th>Thermoeconomics</th>
</tr>
</thead>
<tbody>
<tr>
<td>First law</td>
<td>The law of conservation of energy states that the total energy of an isolated system is constant; energy can be transformed from one form to another, but cannot be created or destroyed.</td>
<td>Profit is a non total differential form that depends on the path of acquisition (i.e. Gaining).</td>
</tr>
<tr>
<td>Second Law</td>
<td>Is an expression of the fact that some heat must <em>always</em> be rejected during a cycle.</td>
<td>The mean capital or standard of living is the integrating factor of profit and leads to the entropy of capital distribution.</td>
</tr>
<tr>
<td>Third Law</td>
<td>The entropy of a perfect crystal at absolute zero is exactly equal to zero. (0 K). Absolute zero is equal to zero on the kelvin temperature scale, which is -273.15°C or -459.7°F.</td>
<td>Work increases capital and reduces capital distribution. (work is related to collecting capital by distributing goods).</td>
</tr>
</tbody>
</table>
OVERVIEW (PHYNANCE)

- Due to its capacity to express comprehensive relationships experienced amid finance as well as physics.
- Highlighted relationships it is evident that individuals get proper understanding concerning the various aspects regarding finance.

- The purpose of facilitating proper understanding promotion concerning various economic aspects.
- Physics knowledge is usually applied towards explanation of experienced economic conditions.

- Economic theories which were traditionally formulated have no capacity to offer substantial explanations regarding experienced conditions hence the need to turn to some theories found within physics which have the capacity to bring about suitable explanations regarding experienced economic situations.

- Phynance has been very essential as it has made it possible for useful knowledge bits to be attained which facilitate substantial understanding regarding various aspects affecting financial markets.
- It is evident that the field has facilitated realization of desirable operations due to the available knowledge bits which steer appropriate solutions attainment regarding experienced problems.
Measure of the energy of a system that is unavailable for doing useful work. In statistical thermodynamics, entropy (usual symbol S) is a measure of the number of microscopic configurations. The interpretation of entropy in statistical mechanics is the measure of uncertainty, or mixedupness.

Entropy is used to help model and represent the degree of uncertainty of a random variable. It is used by financial analysts and market technicians to determine the chances of a specific type of behavior by a security or market.
Entropy (Calculation itself) can be applied and represented by a variable to eliminate the randomness created by the underlying security or asset, which allows the analyst to isolate the price of the derivative.

The best variable is the one that deviates the least from physical reality. In finance, this can be represented with the use of probabilities and expected values.

In other words, entropy is used as a way to identify the best variable for which to define risk within a given system or financial instrument arrangement.

For example, in financial derivatives, entropy is used as a way to identify and minimize risk.

While the calculation itself is evolving, the purpose is clear; analysts are using the concept to find a better way to price complex financial instruments.

Originally used in Thermodynamics and its concepts and relevant principles have been applied to the field of finance for a long period of time.

Its unique advantages in measuring risk and describing distributions. As a result, the applications of entropy in finance are important.

There are many different theories about the best way to apply the concept in financial markets and computational finance.

For example, in financial derivatives, entropy is used as a way to identify and minimize risk.
CONCEPT OF ENTROPY IN ECONOMICS SYSTEMS

Thermodynamics is a statistical theory for large systems, which is actually based on two corresponding concepts:

Based on the Second law of thermodynamics

\[ \delta W = dY - Td\ln P \]

In economics work reduces the entropy of capital distribution: business collects capital from customers by selling goods. This may be repeated by economic Carnot cycles and leads to economic growth. This is Eq leads “macro-economic”, to production of industrial goods and monetary cycles. Work in thermodynamics and production in economics are the same.
According to free energy concept is based on Lagrange principle of statistics. Probability \( P \) is maximized under the constraint \( Y \), \( T \) is the Lagrange parameter.

\[
L = T \ln P - Y \rightarrow \text{max}
\]

In thermodynamics \( L \) is the free energy, \( Y \) is the energy of atomic bonds, \( T \) is the mean kinetic energy or temperature.

This concept may be translated to socio-economic systems with constraints and leads to "micro-economics".

Economy is a market with traders under the constraint of prices. Society is a system of social agents under the constraints of social bonds.

Statistical laws this Eq are never concerned with the type of object, but only with the number of objects.

For this reason it will be necessary to discuss new meanings for the functions \( L, T, P \) and \( Y \) in social and economic systems.

In atomic systems all thermal properties of materials (solids, liquids or gases) may be derived from the Lagrange principle, and it is the object of this section to investigate, whether this concepts is also valid in other systems like economics, social science and politics.
To draw a cycle or a closed integral requires at least two dimensions, x and y. The closed line integral has the general form:

\[ \int a(x, y)dx + b(x, y)dy \]

The two dimensional differential forms are the total differential forms, the function \( f \) exists and is given by the limits of integral, if

\[ \frac{\partial a(x, y)}{\partial y} = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial b(x, y)}{\partial x} \]

Since \( df(x, y) \) depends on the limits of the integral, only the closed integral is zero.

And another two dimensional differential forms

\[ \delta \omega = a(x, y)dx + b(x, y)dy \]

\[ \frac{\partial a(x, y)}{\partial y} \neq \frac{\partial b(x, y)}{\partial x} \]

are non total differential forms, the function \( \omega \) depends on the path of integration, if

Since the integral depends on the path of integration, the closed integral is generally not zero,

\[ \int \delta \omega \neq 0 \]
A two-dimensional non total differential form $\delta w$ can be transformed into a total differential form $df$ by an integrating factor $1/y$. The closed integral will be zero.

According to the first law of thermodynamics states that heat $dQ$ is a non total differential, the closed integral is not zero and the value of $Q$ depends on the path of integration.

$\int \delta Q \neq 0$

The integrating factor $1/y$ leads to a total differential form $df$ of a new function $f$. In thermodynamics the integrating factor $1/T$ leads to the new function $S$, which is called entropy. The closed integral of entropy is zero. The closed integral of heat may now be written in terms of entropy:

$\int \frac{\delta Q}{T}$
\[ \oint \delta Q = \oint TdS \neq 0 \]

This is closed line integral Eq. leads to profit,

\[ \Delta Q = \oint TdS = \oint ydx \neq 0 \]

In thermodynamics entropy is closely connected to the probability (P) of energy distribution in a system like a gas

\[ S = \ln P \]

\[ P = \frac{N!}{(N_1!N_2!...N_k!)/K^N} \]

In economic systems the entropy is closely connected to the capital distribution in an economic system like a market.
### INTERPRETATION OF ENTROPY IN FINANCE

<table>
<thead>
<tr>
<th>1</th>
<th>Entropy is a well-defined quantity in physics that is associated with unused energy (e.g., thermal radiation). It can also be defined as a quantitative representation of “disorder” (or measure of uncertainty).</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>In this context, entropy is viewed as a relationship between macroscopic and microscopic quantities that describes the dispersion of energy. In finance, this relationship can be viewed as to how the number of states (or regimes) in a market translate themselves into a probability distribution of the aggregate market sentiment.</td>
</tr>
<tr>
<td>3</td>
<td>In particular, the concept of entropy (in many different existing formulations) has been extensively used in finance to quantify the diversity and regularity of movements in price across a variety of markets (i.e. stock, currency, future, and commodity).</td>
</tr>
<tr>
<td>4</td>
<td>In physics, the thermodynamic entropy of a macro state (defined by specifying pressure, volume, energy, etc.) is essentially the logarithm of the number of microstates (quantum states) consistent with it; i.e., the number of ways the macro state can be realized.</td>
</tr>
<tr>
<td>5</td>
<td>The main objective of this part is to highlight the interpretation of entropy in the study of financial markets. Focus is emphasized on the application of Shannon entropy measures on the extensive Financial Markets.</td>
</tr>
</tbody>
</table>
Likewise, the economy entropy $S$ to which we refer is a function

$$S(X, Y, Z...) = \log \mathcal{W}(X, Y, Z...)$$

Macroeconomic variables of our theory recognizes

Let $X$ be the discrete random variable of finite range $x_1, \ldots, x_n$ and $p_i$ the probability of $X$ assuming the value $x_i$. The probability obey $p_i \geq 0 (i = 1, \ldots, n)$ and $\sum_{i=1}^{n} p_i = 1$

Shannon’s entropy is then defined with an arbitrary base

$$S(X) = - \sum_{i=1}^{n} p_i \log p_i$$

Entropy reaches a minimum $S=0$ when $X$ is constant and characterized by a fully localized probability distribution $p(x_0) = 1$ and $p(x) = 0$ for all $x \neq x_0$

In contrast, entropy is maximum $S = \log n$ for uniform distributions, where all instance $x_i$ are equally probable. Shannon entropy can also be defined for a continuous random variable $Z$ as follows.

Let $f(z)$ be the probability density function of $Z$, where $f(z) \geq 0$ and $\int f(z)dz = 1$. 
The entropy (Shannon is then defined as

\[ S(Z) = -\int f(Z) \log f(z) \, dz \]

And it is follows rules to the discrete case.

When dealing with continuous probability distributions, a density function is evaluated at all values of the argument. Given a continuous probability distribution with a density function \( f(x) \), then we define its entropy

\[ H = -\int_{-\infty}^{+\infty} f(x) \ln f(x) \, dx \]

\[ \int_{-\infty}^{+\infty} f(x) \, dx = 1 \quad f(x) \geq 0 \]

\[ \sum_{i=1}^{n} p_i = 1, p_i \geq 0 \]

\[ 0 \ln 0 = 0 \]
ENTROPY AND MAXIMIZATION

1. In standard economic utility theory, consumers, faced with a limited income or budget which they can spend, are imagined to choose a particular consumption bundle from a range of possible bundles of goods, services and opportunities at particular prices, such that their perceived utility $Y$ is maximized over time in some manner.

2. Utility decisions of consumers encompass risk, social factors and value.

3. A time element is involved, both in relation to the point at which purchases can be initiated, and the period over which benefits are spread.

4. As consumers proceed through life, each has a developing process that governs their decision-making concerning the opportunities that present themselves, with the nearer-term being more knowable than the longer term.

5. In thermodynamics, entropy is a property that measures the amount of energy in a physical system that cannot be used to do work.
   
   In statistical mechanics it is defined as a measure of the probability that a system would be in such a state, which is usually referred to as the "disorder" or "randomness" present in a system.
Given that systems are not in general reversible then, following whatever means are applied to return a system to its starting point, the net change in cycle entropy is commonly stated as:

\[ \int \frac{dQ}{T} \geq 0 \]

As per the Second Law of thermodynamics, it is impossible to construct a system which will operate in a cycle, extract heat from a reservoir and do an equivalent amount of work on the surroundings. Entropy tends to rise. It is a measure of dispersed value. And the for a reversible process the first law of thermodynamics is state that:

\[ dQ - PdV = dU \]

And closed reversible thermodynamics system there exists a property \( S \), such that a changes in its value between two states is equal is equal to:

\[ S_2 - S_1 = \int_{1}^{2} \left( \frac{dQ}{T} \right)_{rev} \]

The property \( S \) is called the Entropy of the system, and the value \( dS \) is the incremental change in entropy. The suffix ‘rev’ is added as a reminder that the relation holds only for a reversible process.

Or in differential form:

\[ dS = \left( \frac{dQ}{T} \right)_{rev} \]

\[ dQ = TdS_{rev} \]
Now, by combining equation for the First law and for the Second law and inserting the term for the incremental work done \( dW = PdV \)

we could also construct an entropy function for an economic system:

\[
TdS = dU + PdV
\]

And in unit stock terms (N=1) we can write:

\[
TdS = du + Pdv
\]

This two equations set out the general relations between the properties and, when integrated, give the change in entropy occurring between two equilibrium states for a reversible process.

It should be noted that entropy change in economic terms is associated with changes in flow of economic value.

In order to represent the potential work value occasioned by the impact of a motive force/utility value initiating increased or decreased activity flow rates of capital stock, labour and resources and converted into a changed product flow rate, resort can be made to two thermodynamic properties known as the Helmholtz Free Energy function \( F \) and the Gibb Free Energy function \( X \).
These functions are common concepts in the thermodynamic analysis of chemical and gas reactions, particularly for closed systems, though there is no reason why they should not be used also for flows of inputs and output per unit of time, in which case we are considering Free Energy per unit of time.

They express the total amount of Exergy (available energy) which can be used up or passed on during a reaction to equilibrium.

In thermodynamic terms they respectively have the formulae:

\[ dF = -(PdV + SdT) \quad \text{and} \quad dX = (VdP - SdT) \]

Where \( P \) is pressure, \( V \) is volume, \( T \) is temperature and \( S \) is entropy.

Economic equivalent, relating Free Value to price/cost \( P \), volume flow \( V \), entropy \( S \) and the index of trading value \( T \).

The Helmholtz function works in terms of partial volumes, whereas the Gibb function works in terms of partial pressures [economic equivalent price]. The choice made here is the Helmholtz function \( F \) though whichever function is used the result in economic terms is essentially the same.
Continuing with the Helmholtz function, the assumption is made that immediately before the actual point of increased/decreased conversion rate of inputs into outputs, no change in the index of trading value $T$ of either inputs or outputs has occurred.

$$PV = P \nu N = NkT \quad or \quad P \nu = kT = \text{Constant}$$

Thus value flow rate $P \nu$ per unit of stock, equal to price $P$ multiplied by specific volume rate $\nu$, is assumed to be constant for both inputs and outputs, irrespective of the number of units in the stock.

The productive content $k$ is of course constant. This equates to the iso-trading process met and the factor $\nu$ for a unit of stock has some similarities to the chemical concept of an ‘activity coefficient’ applying to the total concentration of stock $N$ available, to equal the effective net input or output in the reaction.
Thus $v = v_N$ as and the factor $v$ was equal to

$$v = \left( \frac{1}{\xi t_1} \right)$$

The ratio of natural lifetime $t_L$ of a unit in a stock, compared to the Std transaction time $t_1$ (usually year).

Returning to our development, because we have assumed $dT = 0$, then for either input or output flows at the system boundary, then the equation can be reduced to:

$$dF = -PdV$$

Which is the negative of the incremental work done $dW$ that we encountered and hence, for a spontaneous reaction to take place to produce additional output flow, consumption (reduction) of free value $F$ occurs, that is $dF$ is negative. By substituting in $PV=NkT$ we have:

$$dF = -NkT\left( \frac{dV}{V} \right)$$
It will be noted that this expression is similar in construction to the Isotrading process, and it will state as:

\[ dS = Nk \left( \frac{dV}{V} \right) \]

Thus a change in free value equates to an opposite change in entropy, adjusted by the index of trading value \( T \).

\[ dF = -TdS \]

By integrating equation above eq, the free value \( F \) inclusive of that for the equilibrium flow volume level of the active output flow and for each of the inactive potential input flows, can be stated as:

\[ F = F^* - NkT \ln(V) \]
MONEY ENTROPY AND INTEREST RATES

1. It is generally accepted among economists that the demand for money is positively related to income and output, but negatively related to interest rates. And it provides the means to keep an economy in balance.

2. However, arguing that the thermodynamic analysis set out so far indicates that economies appear to operate with a polytrophic relationship between specific price, output volume and velocity of circulation, with interrelating and continually changing elasticity.

3. We are also arguing that there is a relationship between the concepts of utility and entropy, and therefore that the utility of money can be represented in terms of entropic value.

4. Thus the inference of the analysis is that interest rates are negatively related to changes in money entropy value.

5. The higher the level of money entropy change, and the higher price inflation, the more negative interest rates have to be to counteract the forces in the economy. Interest is therefore a form of value flow constraint and negative entropy change.
The following relationship might therefore be posited between money balances $N$, interest rates $i$, the velocity of circulation $T$ and entropy change $ds$:

$$\frac{dN}{N} = f\left(i, \frac{dT}{T}, ds_{money}\right)$$

which grows over time according to the level of interest rates.

W. R. T. $i$ Cumulative interest index value $I_t = I_0 e^{it}$

Then for a constant interest rate $i$, the current cumulative index value $I_t$ is calculated from a progression of variable interest rates.

However, interest rates do vary over time according to economic conditions and therefore our cumulative index of interest value $I_t$ is calculated from a progression of variable interest rates.
\[ I_t = I_0(1+i_1)(1+i_2)\ldots(1+i_t) = I_0 \prod_{0}^{t}(1+i_t) \]

Represents a convenient starting point

Although money balances \( N \) exist primarily to facilitate flow of value within an economy, it is not unreasonable to accept that when they reside in a deposit account they will accumulate interest, as would money lent to borrowers accumulate chargeable interest.

Such interest, when payable or chargeable, is included in the total of money balances.

Thus there is likely to be a relationship between our cumulative interest index value \( I \) and both output value \( G \) and money balances \( N \), depending upon the active use in economic output value, and the inactive time spent on balance (which is a function of the velocity of circulation \( T \)).

It will be appreciated of course that with high inflation the index \( I \) is likely to rise significantly with larger interest rates implied. Likewise the number of money instruments \( N \) tends to rise as the currency is depreciated, and output value rises as prices and inflation escalate.
The relationships between the cumulative interest index $I$, output value $G$ (equal to price level $P \times$ output volume $V$), and nominal money balances $N$, according to the equation:

$$\left( \frac{I_2}{I_1} \right) = \left( \frac{G_2}{G_1} \right) = \left( \frac{N_2}{N_1} \right) \text{ or } \frac{dI}{I} = \theta \frac{dG}{G} = \tau \frac{dN}{N}$$

Where the elastic indices $\theta$ and $\tau$ can be variable, and where the third interrelationship is the velocity of circulation $T$, equal to $G/N$, shown earlier in this chapter.

It was assumed that the cumulative index of interest value $I_0$ at the starting point of the time series for each of the two economies was equal to 1.

Though the regression coefficients of the relationships are quite high, there are significance differences in the slope of the curves over time, and hence the elasticity between the functions, as was the case also with the elastic index $n$ between specific price and output volume.
On balance, it appears that the relationship of interest rates to change in output value is stronger than that to money balances.

The production function, the following relationship is proposed to connect interest rates to both output $G$ and entropy $s$: “

“In an economic system, the difference between the rate of change in output value flow $G$ and the rate of change in the Index of Money Interest $I$ is a function of changes in money entropy generated or consumed.”

The expression as

$$ds_{money} = f \left[ k_{money} \left( \frac{dG}{G} - \frac{dI}{I} \right) \right]$$

where $dI/I$ is the short-term interest rate $i$ at any point in time. And $k$ is the assumed ‘productive content’ of money.

The inference of this equation therefore is that an economic system will tend to operate with an entropic difference, positive when expanding and negative when contracting.

A nil value would occur when output value rate of change is matched by interest rates.

A highly positive value would equate to an economy seeking to maximize expansion, without being checked by interest rates. A negative value would equate to high interest rates coupled with low or negative output value growth.
Returning to our money analysis, if further we assume that the productive content $k$ of equation (above) is equal to unity (£1, $1 etc), and the elastic factor $\theta$ at equation (previous) is absorbed into the entropy change $ds$, then above equation becomes:

$$ds_{money} = \frac{dG}{G} - \frac{dI}{I}$$

In this equation the rate of change in the cumulative interest index $I$ is a function of the rate of change in output value $G$, but is negatively related to change in money entropy. Further, by substituting in the general money equation and we get:

$$\frac{dG}{G} = \left( \frac{dP}{P} + \frac{dV}{V} \right) = \left( \frac{dN}{N} + \frac{dT}{T} \right)$$

We can derive a number of other relationships, of which the following is an example:

$$\frac{dN}{N} = \left( \frac{dI}{I} - \frac{dT}{T} + ds_{money} \right)$$

$$\left( \frac{dN}{N} - \frac{dI}{I} \right) = \left( ds_{money} - \frac{dT}{T} \right)$$
It will be noted that this equation also has the same format as in our initial money hypothesis set out at first equation.

Thus the rate of change in money supply is equated to interest rates less the rate of change in the velocity of circulation plus the entropy change.

The relationships involving output value change $\frac{dG}{G}$ appear to work better than with money balances, and this may have something to do with the way money supply has developed, veering away from a flat relationship with output value, with changes in velocity of circulation.

Because of the ‘noise’ inherent in the data, technical changes in the value capacity coefficient $\omega$ (and hence changes in long-run velocity of circulation), and changes in the impact of interest rates on an economy it is inevitable that there will significant deviations between values on either side of each equation.

Thus correction for the biases would improve the correlation of the initial result significantly.

Possible improvements may include better long-run modelling of the value capacity coefficient $\omega$ to estimate entropy change.

Accepting that there is a link between entropy change, output value and interest rates, the key indicator of portending change in an economy will be when sudden changes in the short-run elastic index $n$ occur and it will be appreciated that economic entropy change, positive or negative, represents a measure of whether an economy is likely to expand or contract.
The factors are different for velocity and volume, though the solution for zero money entropy change is still the same.

\[ n = \gamma = \frac{\omega + 1}{\omega} \]

\[ ds_{money} = \left( \omega + \frac{1}{1 - n} \right) \frac{dT}{T} \]

\[ ds_{money} = (\omega - \omega n + 1) \frac{dV}{V} \]

\[ \frac{dT}{T} = (1 - n) \frac{dV}{V} \]

Thus the condition for positive money entropy change is therefore: Factor and multiplicand at this equations are either ‘both positive’ and ‘both negative’. And the condition for negative money entropy change is: Factor and multiplicand at this equations must be opposite in sign i.e. factor positive and multiplicand negative, and vice versa.

\[ \omega \frac{dT}{T} = - \frac{dV}{V} \]
Entropy is a measure of randomness. Much like the concept of infinity, entropy is used to help model and represent the degree of uncertainty of a random variable.

It is used by financial analysts and market technicians to determine the chances of a specific type of behavior by a security or market.

Although the word entropy was originally used in thermodynamics, its concepts and relevant principles have been applied to the field of finance for a long period of time. Entropy has its unique advantages in measuring risk and describing distributions.

This paper reviewed representative work on how the applications of entropy in finance, mainly in financial markets and money entropy and interest rates.
THANK YOU