Abstract

This article reconstructs the history of monetary policy of the central bank of Colombia in the period 1990 to 2010 in which explicit inflation targeting was adopted by October of 2000. To do so we developed theoretically a modified Taylor rule with interest rate smoothing for an open and small economy and accordingly estimate a two regime Markov switching model which allows the switching dates to be endogenously determined. We find that one regime had explicit inflation targeting (from the year 2000 up to 2010) in which the inflation rate is a stationary series, given that the central bank enforced a monetary policy that satisfied the Taylor principle. This inflation stabilizing regime did show up in some quarters before the year 2000 but was not the predominant. The other regime was the more prevalent during the 1990s but did not satisfy the Taylor principle allowing a unit root behavior of the inflation rate. Moreover we find that the central bank reacted aggressively during the 1990s to output fluctuations while having an accommodating behavior for this variable during explicit inflation targeting from 2000 onwards.

JEL Code: E52, E58
Key words: Taylor rule, Taylor principle, Markov Switching Model
RECONSTRUYENDO LA RECIENTE HISTORIA DE LA POLÍTICA MONETARIA DE COLOMBIA DESE 1990 A 2010

Andrés Felipe Giraldo
Departamento de Economía
Pontificia Universidad Javeriana

Martha Misas
Departamento de Economía
Pontificia Universidad Javeriana

Edgar Villa
Departamento de Economía
Pontificia Universidad Javeriana

Resumen

Este artículo reconstruye la historia de la política monetaria del Banco de la República de Colombia en el período 1990 a 2010 durante el cual inflación objetivo explícito se adoptó por parte del banco en Octubre de 2000. Para ello se desarrolla teóricamente una regla modificada de Taylor con suavizamiento de tasa de interés para una economía pequeña y abierta y luego se estima un modelo de Markov de dos regímenes que permite determinar endógenamente las fechas de cambio de régimen. Encontramos en un régimen (predominante de 2000 a 2010) que la tasa de inflación es una serie estacionaria dado que el Banco de la República implementó una política compatible con el principio de Taylor. Aunque este régimen estabilizador de la inflación ocurrió algunos trimestres antes del año 2000 no fue el régimen predominante. El otro régimen fue predominante durante los años 90 pero no fue compatible con el principio de Taylor lo que generó un comportamiento persistente de raíz unitaria de la tasa de inflación. Más aún, encontramos que el Banco de la República reaccionó agresivamente durante los año 90 a fluctuaciones del producto mientras que tuvo un comportamiento acomodaticio para esta variable después del 2000.

Código JEL: E52, E58
Palabras clave: Regla de Taylor, Principio de Taylor, Modelo Markov de Regímenes
1 Introduction

Since the debate on discretion versus monetary rules (Kydland and Prescott (1977), Barro and Gordon (1983)) there has been an emerging consensus in monetary economics on the convenience for an economy that its central bank pursue price stabilization under a transparent strategy in which credible information is revealed to the public in a systematic way. This has generated a systematic research to develop strategies that would solve the time inconsistency problem of monetary policy. An emerging theme since the 80’s has been that of generating simple monetary rules that would convey transparent information to the public in an accessible way which has generated a search for implementing in an operational way simple strategies for monetary policy. The first of these strategies that was proposed was that of monitoring monetary aggregates following the work of M. Friedman (1968) in order to control inflation (monetarism). The second strategy proposed was that of targeting the exchange rate in order to avoid independent monetary policy and import inflation from the country that was used to anchor the currency of the local economy. The third strategy proposed was that of explicit inflation targeting as a way of controlling inflation (Bernanke and Mishkin (1997)). Behind all these approaches there has been an intermediate goal of monetary policy that seeks to relate a variable controlled by the central bank to affect indirectly inflation, which has been called the nominal anchor. In the first approach the idea is to use the monetary aggregates as the nominal anchor to control inflation, while in the second approach the nominal anchor proposed is the nominal exchange. Finally, explicit inflation targeting has a focus on inflation forecasts and therefore the nominal anchor is the intervention interest rate. Here is where Taylor rules have emerged. Since the publication of John Taylor’s seminal paper about the practice of monetary policy (Taylor, 1993) it has become frequent the specification and estimation of a reaction function that characterizes the behavior of a central bank in order to obtain a simple guide regarding how to intervene in the economy. The goal of the Taylor rule is to summarize into a simple rule the complex and challenging process of monetary policy decisions. Importantly, most Taylor rules in the literature are thought for the United States economy which is characterize by being a close economy. In contrast in this article we develop a modified Taylor rule that incorporates both interest rate smoothing and exchange rate targeting for an open and small economy. We then compare this modified Taylor rule with the original Taylor rule and show that it also satisfies the Taylor principle under certain restrictions.

We briefly introduce as a preface of our theoretical framework the history of monetary

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1We would like to thank Banco de la República for generously providing the data used in this study. We also thank Adolfo Cobo for his guidance in choosing the series to be used in this project. Finally, we thank Hernando Vargas and Andrés González for their useful comments on a preliminary draft. Evidently all remaining errors are our own. Comments are welcomed and could be sent to a.giraldo@javeriana.edu.co, mmisas@javeriana.edu.co or e.villa@javeriana.edu.co.
policy in Colombia since the independence of the central bank (Banco de la República) of Colombia from the government by constitutional mandate in 1991. Our reconstruction is that of the monetary policy history of Banco de la República from 1990 to 2010. Importantly, Banco de la República announced the implementation of explicit inflation targeting in October of 2000 which presents itself as an exogenous event that could justify splitting the considered data in two periods: one subsample before and one after October of 2000 and then analyze the Taylor rule "as if" the central bank’s behavior could be modeled in this way in both periods. Nonetheless, we refrain from taking such an adhoc position to model a two regime model. We prefer to estimate our modified Taylor rule in a two regime Markov model that endogenously allows switching dates to occur in a probabilistic way and not deterministically as the previous approach would suggest. We also consider a Markov switching unit root test for the inflation rate in order to test whether one could have stationarity in the regime that satisfies the Taylor principle while not having a unit root in the regime that does not. Our results go in line with the monetary history of the central bank of Colombia in several ways. We find that one regime had explicit inflation targeting (from the year 2000 up to 2010) in which the inflation rate is a stationary series since the central bank enforced a monetary policy that satisfied the Taylor principle. Even though this inflation stabilizing regime showed up in some quarters before the year 2000 the other regime was actually the predominant one during the 1990s and did not satisfy the Taylor principle allowing a unit root behavior of the inflation rate. Moreover, we find that the central bank reacted aggressively during 1990s to output fluctuations while having an accommodating behavior for this variable during explicit inflation targeting from 2000 onwards.

The article is organized such that in the first part we present a brief monetary history of the central bank since its independence from the government by constitutional mandate in 1991. We then present a literature review on Taylor rules. We then introduce the model in which we derive optimally the modified Taylor rule with interest rate smoothing for a small and open economy. We analyze the rule and compare it with the original Taylor rule as well as verifying if it satisfies the Taylor principle. Next we present the data to be used and then the empirical framework in which we present the two regime Markov switching model. We then present our results for the two regime Markov unit root test for the inflation rate and then the results for the two regime Markov switching model. Finally we then present our conclusions.
2 Brief History of Monetary Policy in Colombia during 1990 to 2010

The proclamation of the Constitution of 1991 allowed the independency of the Colombian central bank from the government and brought with it operative changes and different procedures in managing the monetary policy of the country\textsuperscript{2}. Before the constitutional change was approved the monetary policy of the Banco de la República was highly dependent on the fiscal policy of the government and actually did not impose stringent restrictions on the emission of currency in order to finance public spending (Kalmanovitz, 2003). Moreover, the monetary policy that followed the bank did not contribute with the financial deepening of capital markets mainly because the intervention interest rate was not really a reflection of market conditions but more a response to political pressures (Sanchez et al., 2007).

The Constitution of 1991 gave Banco de la República the mandate to maintain price stability in line with other economic policy objectives and to achieve this it conferred to the bank monetary, foreign exchange and credit instruments. Up to this day all decisions are taken by a board of directors which has the responsibility to fulfill the constitutional mandate. During the first years after 1991 the bank had the traditional dichotomy of designing the monetary policy strategy: either targeting monetary aggregates or targeting an intervention interest rate (Hernández and Tolosa, 2001). The importance of this decision was related with the choice of an adequate nominal anchor in order to send the right signals to the markets\textsuperscript{3}.

According to Hernández and Tolosa (2001) the Colombian central bank chose to target monetary aggregates. The main intermediate monetary target chosen was M3 (which includes monetary and some non monetary liabilities of the bank) because the recommended empirical instruments allowed some adequate monitoring, given its relationship with the final target: the inflation rate. Besides tracking this monetary aggregate the board of Banco de la República during the nineteen nineties set up an exchange rate band where it expected the nominal exchange rate to remain within\textsuperscript{4}. This policy required the central bank to make an intervention when the exchange rate hit the top or the bottom of the maintained band. In both cases the monetary policy became dependent on the foreign exchange rate policy and

\textsuperscript{2}There were more structural changes such as a greater financial liberation, a different exchange rate system among many others. In this sense the changes were not only institutional.

\textsuperscript{3}An introductory review about the role of a central bank can be seen in Mishkin (2007, chapter 2).

\textsuperscript{4}With the exchange rate band, the central bank expected to provide a credible market signal about expectations which were coherent with the inflation target. The bank did not have in its monetary policy strategy the intention of keeping a fixed exchange rate regime but neither a highly volatile but flexible exchange rate regime. In practice, this exchange rate band had in some sense the best and worst of both exchange rate regimes. For example, when Colombia was exposed to a speculative attack on its local currency (peso), the central bank abandoned the interest rate band and aggregate monetary bands in order to maintain the exchange rate band. Nonetheless the exchange rate band disappeared in September of 1999. Since this moment onwards the exchange rate regime has a controlled flexibility.
it seemed that the ultimate target, the inflation rate, was moved to a second level priority.

During the nineteen nineties Banco de la República had a band for its intervention nominal interest rate. The objective was for the money market to understand the strategy in order to achieve the maintained target as well as the procedure to accomplish the ultimate inflation rate. During those years the main monetary policy instruments were open market operations used to increase or reduce the liquidity in the monetary market having as a target the M3 growth rate. It could be interpreted that the central bank had three (intermediate) objectives in those years: M3, the exchange rate and the interest rate, all were used in order to reach a final target which was of course the inflation rate. Notwithstanding, to manage these three objectives is not free of a conflictive nature.

Monetary policy during the nineteen nineties seemed complicated under other structural reforms that were done at the beginning of the nineties and which coincided with the independency of Banco de la República. Among the structural reforms that were generated during those years was greater openness of capital markets, greater liberalization of the financial system and the decentralization of fiscal policy within the departments of Colombia. Actually the board of directors defined the order of priorities in case the three policy objectives were in conflict: the first objective was to drop the interest rate band and the other two were analyzed depending on the current economic shocks (Hernández and Tolosa, 2001, pp. 5).

In October of 2000, the board of directors announced that Banco de la República would adopt an inflation targeting policy in order to maintain the stability of prices. Since this moment onwards and up to this day the central bank changed its main instrument for the intervention interest rate and accorded with others countries that also adopted inflation targeting as the main monetary policy. The idea was to make monetary policy more transparent and more easily understood for the public as well as for the markets. According to the literature on inflation targeting, this regime is characterized by three factors: 1) an announcement of the numerical inflation target (or the range where the long run target was to be realized); 2) an increased importance of the role to forecast inflation; and 3) a consistent and systematic strategy of communication to the public to increase transparency of the policy.

Although the central bank only adopted inflation targeting in 2000 there was a special feature of the policy followed by Banco de la República since the adoption of the new Constitution in 1991: the explicit announcement of a quantitative inflation target. Even so

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5 As it is known monetary policy has a trilemma: when a central bank wishes to achieve three targets related to a determined exchange rate, a targeted interest rate and free mobility of capital there is a problem to achieve the three objectives at the same time. The decision around these objectives and the restrictions involved the procedure to achieve the targets is summarized by Gómez (2006).

6 In order to capture the forward-looking characteristics of monetary policy (Mishkin, 2007, pp. 39).

7 For a detailed treatment of the inflation targeting as a monetary policy regime see Svensson (2008, 2011).
the reviewed history of the monetary policy followed by Banco de la República suggests two distinct periods: one from 1991 to 2000 where the regime that was used had *implicitly* an inflation target and another period from 2000 up to the present where the regime uses an *explicit* inflation target. Accordingly the instruments used to implement the monetary policy can be identified in these two periods as follows: the use of monetary aggregates since the independence of the bank through 1998, then the use of an exchange rate regime\(^8\) that lasted until a speculative attack on the currency which made Banco de la República abandon it by 1999, and finally the use of an interest rate target since the explicit adoption of inflation targeting by the central bank from 2000 up to the present. This suggests an hypothetical two distinct periods in which monetary policy was done differently by Banco de la República: from 1991 to 2000 and 2000 up to 2010.

### 3 Literature Review

The subject regarding monetary policy rules comes out from the rules versus discretion debate and the rise of inflationary bias due to time inconsistency of a central bank\(^9\). To reach time consistency monetary theory has generated a big body of literature where it states the importance of monetary policy rules. As Woodford (2003a) argues:

> “...there is a good reason for a central bank to commit itself to a systematic approach to policy that not only provides an explicit framework for decision making within the bank, but that is also used to explain the bank’s decisions to the public...” (Woodford, 2003a, pp. 14)

The literature on inflation targeting argues that if the central bank of a country acts without a systematic (strict or flexible\(^{10}\)) rule, only using discretion, it is likely that the results of its monetary policy would end up being suboptimal, regardless whether it has the best human resources and technical tools available. Given this the proponents of inflation targeting argue for the use of an explicit rule for a central bank in order to guide its monetary policy. Within the proponents Mishkin (1999) has shown different monetary regimes available to conduct monetary policy where the common denominator of these regimes is the existence of a nominal anchor in order to control the expectations of the agents in the economy and increase the effectiveness of the policy. Mishkin identifies three anchors depending on the regime: monetary aggregate regimes, exchange rate regimes and inflation targeting

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8 As Bernal (2002) argues Banco de la República did have some space for changes in the intervention interest rate but only when the exchange rate was close to the two extremes of the exchange rate band.

9 Walsh (2003) presents a detailed discussion about this topic of monetary policy.

10 See (Walsh, 2003, chapter 7) for a detailed discussion.
Each of them is characterized by the use of an intermediate target that allows to anchor the expectations and reach the ultimate goal of any monetary policy, the inflation rate.\footnote{Actually, Mishkin (1999) identifies a fourth regime, which is called "just do it", and argues that this regime is used in United States. However, this regime has an implicit rather than explicit nominal anchor.}

Although the central bank can summarize some important issues of its monetary policy into a (simple or complicated) monetary policy rule, the lags of any monetary policy makes more difficult the process of taking adequate decisions. This is a reason why the monetary policy should have a forward looking component (Mishkin, 2007, chapter 2). Since the publication of John Taylor’s seminal paper about the practice of monetary policy (Taylor, 1993) it has become frequent the specification and estimation of a reaction function that characterizes the behavior of a central bank in order to obtain a simple guide regarding how to intervene in the economy. The goal of the Taylor rule is to summarize into a simple rule the complex and challenging process of monetary policy decisions. The original rule stated by Taylor, for the Federal Reserve of the United States economy, is actually a static or contemporaneous rule which is specified as follows

$$i_t = \pi_t + 0.5x_t + 0.5(\pi_t - 2) + 2$$

where $i$ is the federal fund rate, $\pi$ is the inflation rate of the last four quarters, 2 is the inflation rate target and $x$ is the output gap (the difference between the logarithm of the observed GDP and the potential GDP). The rationale of the rule is as follows: if the output gap is positive the Fed responds increasing the interest rate to contain the inflationary pressures while if the inflation is increasing the federal funds rate is also increased. The rule suggests that if $x$ is zero and $\pi$ is 2 the implicit real interest rate implied is 4. Taylor showed that this rule describes well the monetary policy of the United States during the late eighties and early nineties.

Now since the publication of Taylor’s paper the area of econometric assessment of monetary policy rules was established as a prolific macroeconomic and public policy area of research. The first objective was to make a quantitative evaluation of the Taylor rule to test if this specification described the true path of the intervention interest rate, at least for the United States. The next step in that research program was to evaluate the performance of different specifications of monetary rules into macroeconomic models and assess its optimal conditions\footnote{Taylor (1999) is a good compilation of many works done in that tradition.}. Taylor’s contribution has resulted in normative and positive implications even though some criticisms have appeared\footnote{One criticism of simple interest rate rules is that, under certain circumstances, they may induce instability (Clarida et al., 2000).}. On the normative side, the rule is in accord with
the main principles for optimal policy that we have described above. In at least an approximate sense, the rule calls for a countercyclical response to demand and accommodation of shocks to potential GDP that do not affect the output gap. On the positive side Taylor’s contribution has showed that under certain parameter values the rule that he proposed does a reasonably good description of monetary policy over the period in which he studied the US economy (period 1987-92).

There are different Taylor rules which have been evaluated using different empirical strategies. Likewise from a theoretical perspective the rule has been explained out of microfoundations. However, it has been more traditional to formulate Taylor type rules from economic intuition without a structural model behind it. In the literature it is common to find a great variety of articles either only on the empirical side or to examine the appropriateness of the rule under simulations and then contrast its optimality and robustness in the same context. One important characteristic of the Taylor rules is the concept related to the so called Taylor Principle which Woodford (2003a) defines it as follows:

“...In Taylor’s discussions of the rule, he places particular stress upon the importance of responding to inflation above the target rate by raising the nominal interest-rate operating target by more than the amount by which inflation exceeds the target...” (Woodford, 2003a, pp. 40)

If the rule is expressed as

\[ i_t = \bar{r} + \pi + \gamma_{\pi} (\pi_t - \bar{\pi}) + \gamma_x x_t \]  

(2)

where \( i \) is the policy or intervention interest rate, \( \bar{r} \) is the real interest rate, \( \pi \) is the inflation target, \( x \) is the output gap, \( \gamma_{\pi} \) is the inflation aversion, \( \gamma_x \) is the business cycles aversion. The Taylor principle is satisﬁed if \( \gamma_{\pi} > 1 \). Some of authors in the tradition on evaluation of Taylor rules have tried to reconstruct the monetary history of certain economies, in the same way as Taylor (1993), estimating the inflation aversion \( \gamma_{\pi} \) coefficient\(^{15}\). As Orphanides (2008) argues there are different sorts of Taylor rules which try to describe the way a central bank adjusts its intervention interest rate to changes in economic activity and inflation.

Among the rich literature on Taylor rules we would like to emphasize two articles that are related to our article. Importantly when a central bank follows a monetary rule compatible with the Taylor principle it exhibits a high inflation aversion and reacts strongly against inflationary pressures. This has been shown empirically by two articles using a Markov switching model: Kuzin (2004) for the Bundesbank and Murray et al (2008) for the Federal

Reserve. Kuzin (2004) estimates a simple backward-looking Taylor rule (with interest rate smoothing) in a time-varying coefficient framework with quarterly German data for the period 1975–1998. The main finding is that the inflation aversion of the Bundesbank was not constant over time and exhibits some sudden and large shifts during the period of monetary targeting. There are phases with low and with high inflation aversion, the former compatible with the Taylor principle while the latter violates it. Kuzin argues that his results provide evidence that the Bundesbank followed the so-called ‘opportunistic approach’ to disinflation.

Murray et al. show that a “textbook” macroeconomic model with an IS curve, a Phillips curve, and a Taylor rule, should exhibit a stationary inflation rate if and only if the central bank obeys the Taylor principle since the real interest rate is increased when inflation rises above the target inflation rate. Moreover, they argue that there is no reason to presume that monetary policy per se is either always stabilizing or always not stabilizing and therefore they suggest that the inflation rate might switch from a stationary regime to a unit root behavior regime depending on whether the policy followed the Taylor principle or not. They estimate a Markov switching model for inflation and show that inflation for the US economy is best characterized by two states, one stationary and the other with a unit root. They find that the unit root state spans most of the period from the 1967–1981 while the stationary state appears since the second quarter of 1981 up to 2008. Consistent with this they also estimate a Markov switching model for various real-time forward looking Taylor rules and find that the considered Taylor rule equation switches between states where the Federal Reserve did and did not try to stabilize inflation by following the Taylor Principle. More precisely, they find that even though the pre and post-Volcker sub-samples each contain multiple Taylor rule regimes, the Volcker tenure at the Federal Reserve, for most of the time did not follow a Taylor rule but did switch to a stabilizing Taylor rule state in 1985:4, which has endured up to 2008.

Furthermore, Murray et al. (2008) contrast their result to Orphanides (2004). Using data from 1965:4 – 1995:4, Orphanides estimates forward looking Taylor rules and splits the sample into pre and post-Volcker periods, with the change occurring between 1979:2 and 1979:3. He concludes that there was no significant change in the Federal Reserve’s response to inflation before and after Volcker: in both regimes the Federal Reserve was estimated to have followed a stabilizing Taylor rule. This is almost exactly the opposite of what Murray et al (2008) find. They argue that Orphanides (2004) splits his sample after 1979:2, which is an intuitive break date, but chosen exogenously and which he imposes to analyze the corresponding two regimes. Murray et al (2008) suggest that when the break date is endogenized, via Markov switching, each of Orphanides’ “regimes” contains periods where the Federal reserve both did and did not follow the Taylor principle. They argue that not only did the Federal Reserve change its response to inflation throughout the entire sample, but that the timing of these changes is not simply pre and post Volcker. Indeed, for the
Volcker years, they conclude that it was not until he had less than two years remaining in his term that monetary policy permanently switched to a stabilizing Taylor rule.

Both Kuzin (2004) and Murray et al. (2008) can be interpreted as warning us of the dangers of imposing exogenously a break date given by economic history. Hence, we do not impose the known date of October 2000 as a break date when Banco de la República announced explicitly inflation targeting as a monetary policy. On the contrary, we endogenize a potential break date in a Markov two regime model and let the empirical model tell us if there are two separate regimes of monetary policy for Colombia which might or might not coincide with what economic history tells us. Nonetheless we do want to study if the behavior of Banco de la República can be associated with a low inflation aversion before October of 2000 and a high inflation aversion afterwards as economic history suggests.

In order to capture different characteristics of the monetary policy rules à la Taylor, it is important to take into account some factors that can influence the decision making process: i) it must capture the desire of stabilizing GDP, ii) it should include interest rate smoothing, iii) it might be important to consider the exchange rate as another variable that may be included in a modified Taylor rule, specially for open and small economies.

One way of capturing output stabilization and an interest rate smoothing preference from the central bank is the following modified Taylor rule as in Kuzin (2004) which can be expressed as:

\[ i_t = \left[ \bar{r} + \bar{\pi} + \gamma_\pi (\pi_t - \bar{\pi}) + \gamma_x x_t \right] (1 - \rho) + \rho i_{t-1} \]  

where \( \rho \) is the parameter that measures the smoothing of the interest rate by the central bank. Furthermore, Woodford (2003a) argues that the reduction of both the interest rate and the inflation variability are appropriate goals of monetary policy and an interest rate smoothing rule allows a greater degree of stabilization of the long-run price level by making inflation fluctuations less persistence (Woodford, 2003a, 98-99). Nonetheless, the functional

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16 Consider \( \gamma_x \) in the equation (2). If there is some mandate for the central bank to stabilize the economy, the way to do it is to include the output gap explicitly into the rule as McCallum (2001) has shown to be desirable. Although it is difficult to observe the gap since potential GDP must be estimated which makes it difficult to evaluate the performance of the rule. However it is important to do an effort in order to estimate the variable because the central bank should stabilize the business cycles (Mishkin, 2007, chapter 4)

17 Although the interest rate inertia was born as an empirical thought idea (Clarida et al., 1998; Woodford, 2003a) Woodford (2003b) shows that under some circumstances it is optimal to include smoothing in the intervention interest rate for a central bank when some economic condition changes. Clarida et al. (1998) state that it is desirable because of the “...fear of disrupting capital markets, loss of credibility from sudden large policy reversals, the need for consensus building to support a policy change, etc” (Clarida et al., 1998).

18 Svensson (2000), Ball (1999) and Taylor (2001) show that the original Taylor rule performs quite poorly in an open economy context which motivates us to include it within our optimal monetary rule.
form in equation (3) is ad hoc and does not come from an optimal program. We will have much to say about this in our structural model developed below which derives optimally the monetary rule with interest rate smoothing.

4 Model

As the literature in optimal monetary rules argues a policy of inflation rate targeting implies optimal rules that take into account both changes in the output gap and the inflation rate. The original Taylor rule includes only these changes in a very simple way presumably because it was appropriate for a big close economy as the United States. Nonetheless, as mentioned above, interest rate smoothing as well as exchange rate targeting could be included in a modified Taylor rule specially for an open and small economy as Colombia. In this section we want to justify theoretically the modified Taylor rule that we end up estimating in a two regime Markov switching model. To do this we develop a structural model that derives the optimal monetary rule for an open economy which yields interest rate smoothing. We then analyze it and compare it with the original Taylor rule for a closed economy as in equation (1).

4.1 Set Up

Consider a central bank of a small open economy that takes as given an intertemporal IS aggregate curve denoted as

\[ x_t = E_t x_{t+1} + \delta x_{t-1} - \sigma [i_t - E_t \pi_{t+1} - r^n] + \alpha_1 [e_t - e^n] + \varepsilon_{1t} \]  

and an aggregate supply curve

\[ \pi_t = k x_t + \beta E_t \pi_{t+1} + \gamma \pi_{t-1} + \alpha_2 [e_t - e^n] + \varepsilon_{2t} \]

where \( x \) denotes the log of the output gap, \( i \) the log of nominal (short run) interest rate, \( \pi \) the log of the inflation rate, \( e \) the log of real (short run) exchange rate, while \( r^n \) and \( e^n \) denote respectively the long run levels of the logs of the real interest rate and real exchange rate which are both assumed to be determined by exogenous factors. The term \( e^n \) reflects an exchange rate target that the central bank might want to maintain. The parameters are assumed to have the following signs: \( \sigma > 0, k > 0, \gamma \in [0, 1), \beta \in (0, 1), \delta > 0 \) while \( \alpha_1 \) and \( \alpha_2 \) could be positive but not necessarily so. The terms \( \varepsilon_{1t} \) and \( \varepsilon_{2t} \) denote respectively the demand and supply disturbances where we assume they are iid normally distributed with mean zero and variances \( \sigma_1^2 \) and \( \sigma_2^2 \) respectively. The operator \( E_t (\cdot) \) denotes the expected value at time \( t \) of the variable next period using all observable information up to time \( t \).
The central bank is assumed to have a loss function represented by

\[
\frac{L_t}{2} = (\pi_t - \bar{\pi})^2 + \lambda_1 \pi_t^2 + \lambda_2 (e_t - e^n)^2 + \lambda_3 (e_t - e_{t-1})^2 + \lambda_4 (i_t - i_{t-1})^2
\]

(6)

where the lambdas are assumed to be positive. The first two terms are common to all monetary rules where \(\lambda_1\) reflects the weight of the output gap in the loss function which is always seen as relative to the weight of one for changes in the inflation rate from a maintained target \(\bar{\pi}\). Now the term associated with \(\lambda_2\) reflects our concern that the central bank of Colombia had an exchange rate band explicitly during the 1990s and this term reflects a preference for deviations of the real exchange rate from the implicit target denoted as \(e^n\). Moreover, we also consider that the central bank could intervene under strong fluctuations of the real exchange rate from one period to another which is summarized by the term associated with \(\lambda_3\). Finally, the fluctuation of the nominal interest rate is captured by the term associated with \(\lambda_4\) in the same way as in Woodford (2003a).

We assume that the behavior of the central bank could be understood as the policy that comes out of minimizing (6) subject to (4) and (5). The corresponding Lagrangian is defined by

\[
\mathcal{L} = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \frac{L_t}{2} + \phi_1 [x_t - x_{t+1} - \delta x_{t-1} + \sigma (i_t - \pi_{t+1} - r^n) - \alpha_1 (e_t - e^n) - \varepsilon_{1t}] + \phi_2 [\pi_t - \beta \pi_{t+1} - \gamma \pi_{t-1} - k x_t - \alpha_2 (e_t - e^n) - \varepsilon_{2t}] \right\} \right\}
\]

where \(\phi_1\) and \(\phi_2\) are the Lagrangian multipliers and which we assume constant through time.\(^{19}\) Since the loss function is quadratic the following first order conditions that solve the optimization program are necessary and sufficient

\[
(\pi_t) : -\beta^{t+1} [\phi_2 \gamma] + \beta^t [\pi_t - \bar{\pi} + \phi_2] - \beta^{t-1} [\sigma \phi_1 + \phi_2 \beta] = 0
\]

(7)

\[
(x_t) : -\beta^{t+1} [\phi_1 \delta] + \beta^t [\lambda_1 x_t + \phi_1 - k \phi_2] - \beta^{t-1} [\phi_1] = 0
\]

(8)

\[
(i_t) : \beta^t [\lambda_4 (i_t - i_{t-1}) + \phi_1 \sigma] - \beta^{t+1} [\lambda_4 (E_t i_{t+1} - i_t)] = 0
\]

(9)

\[
(e_t) : \beta^t [\lambda_2 (e_t - e^n) + \lambda_3 (e_t - e_{t-1}) - \phi_1 \alpha_1 - \phi_2 \alpha_2] - \beta^{t+1} [\lambda_3 (E_t e_{t+1} - e_t)] = 0
\]

(10)

From (7) we get

\[
\phi_2 = \frac{(\pi_t - \bar{\pi}) \beta - \sigma \phi_1}{\beta^2 \gamma}
\]

(11)

while from (8) we obtain

\[
\phi_2 = \frac{\phi_1 [1 - \beta \delta - \beta^{-1}] + \lambda_1 x_t}{k}.
\]

\(^{19}\)This constancy of the Lagrange multipliers is a restriction since Woodford (2003a) does not impose it. Nonetheless we do it to make the analysis tractable analytically since otherwise we would have had to resort to simulations.
Equalizing these last two equations yields

\[
\begin{align*}
(\pi_t - \bar{\pi}) \beta k - \sigma k \phi_1 &= \beta^2 \gamma \phi_1 \left[ 1 - \beta \delta - \beta^{-1} \right] + \beta^2 \gamma \lambda_1 x_t \\
(\pi_t - \bar{\pi}) \beta k - \beta^2 \gamma \lambda_1 x_t &= \phi_1 \left( \beta^2 \gamma \left[ 1 - \beta \delta - \beta^{-1} \right] + \sigma k \right) \\
\phi_1 &= \frac{(\pi_t - \bar{\pi}) \beta k - \beta^2 \gamma \lambda_1 x_t}{\beta^2 \gamma \left[ 1 - \beta \delta - \beta^{-1} \right] + \sigma k} \\
&\equiv m_1 (\pi_t - \bar{\pi}) + n_1 x_t.
\end{align*}
\]

where the \( m_1 \) and \( n_1 \) are functions of parameters given by

\[
\begin{align*}
m_1 &= \frac{\beta k}{\beta^2 \gamma \left[ 1 - \beta \delta - \beta^{-1} \right] + \sigma k}; \\
n_1 &= \frac{-\beta^2 \gamma \lambda_1}{\beta^2 \gamma \left[ 1 - \beta \delta - \beta^{-1} \right] + \sigma k}.
\end{align*}
\]

Note that in general \( \phi_1 \neq 0 \) unless \( \pi_t + \frac{m_1}{n_1} x_t = \bar{\pi} \) which we rule out by assumption. Equation (12) replaced in (11) gives us

\[
\phi_2 = \frac{(\pi_t - \bar{\pi}) \beta - \sigma \left( \frac{(\pi_t - \bar{\pi}) \beta k - \beta^2 \gamma \lambda_1 x_t}{\beta^2 \gamma \left[ 1 - \beta \delta - \beta^{-1} \right] + \sigma k} \right)}{\beta^2 \gamma} 
\]

\[
= m_2 (\pi_t - \bar{\pi}) + n_2 x_t.
\]

where

\[
\begin{align*}
m_2 &= \frac{\beta \left[ 1 - \beta \delta - \beta^{-1} \right]}{\beta^2 \gamma \left[ 1 - \beta \delta - \beta^{-1} \right] + \sigma k}; \\
n_2 &= \frac{\sigma \lambda_1}{\beta^2 \gamma \left[ 1 - \beta \delta - \beta^{-1} \right] + \sigma k}.
\end{align*}
\]

Note again that \( \phi_2 \neq 0 \) unless \( \pi_t - \frac{n_2}{m_2} x_t = \bar{\pi} \) which again we rule out by assumption. Now from equation (9) we get

\[
\phi_1 = \frac{\beta \left[ \lambda_4 \left( E_i i_{t+1} - i_t \right) \right] - \lambda_4 \left( i_t - i_{t-1} \right)}{\sigma}
\]

which equalized with (12) yields

\[
m_1 (\pi_t - \bar{\pi}) + n_1 x_t = \frac{\beta \lambda_4 E_i i_{t+1} - \lambda_4 \left( 1 + \beta \right) i_t - \lambda_4 i_{t-1}}{\sigma}
\]

which rearranged and lagged one period gives us

\[
\pi_{t-1} - \bar{\pi} = \frac{\beta \lambda_4}{\sigma m_1} i_t - \frac{\lambda_4}{\sigma m_1} \left( 1 + \beta \right) i_{t-1} - \frac{\lambda_4}{\sigma m_1} i_{t-2} - \frac{n_1}{m_1} x_{t-1}.
\]

While from equation (10) we get

\[
\lambda_2 \left( e_t - e^n \right) + \lambda_3 \left( e_t - e_{t-1} \right) - \phi_1 \alpha_1 - \phi_2 \alpha_2 - \beta \lambda_3 E_t e_{t+1} + \beta \lambda_3 e_t = 0
\]
and using (12) and (14) to replace \( \phi_1 \) and \( \phi_2 \) allows us to obtain
\[
[\lambda_2 + \lambda_3 (1 + \beta)] e_t = \lambda_2 e^n + \lambda_3 e_{t-1} + \beta \lambda_3 E_t e_{t+1} + \alpha_1 \phi_1 + \alpha_2 \phi_2 \\
= \lambda_2 e^n + \lambda_3 e_{t-1} + \beta \lambda_3 E_t e_{t+1} \\
+ (\alpha_1 m_1 + \alpha_2 m_2) (\pi_t - \bar{\pi}) + (\alpha_1 n_1 + \alpha_2 n_2) x_t
\]
or equivalently when lagged one period and rearranged
\[
e_t = \frac{\lambda_2 + \lambda_3 (1 + \beta)}{\beta \lambda_3} e_{t-1} - \frac{\lambda_2}{\beta \lambda_3} e^n - \frac{1}{\beta} e_{t-2} \\
- \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \lambda_3} (\pi_{t-1} - \bar{\pi}) - \frac{\alpha_1 n_1 + \alpha_2 n_2}{\beta \lambda_3} x_{t-1}.
\] (17)

Replacing (17) in (16) one gets a simple expression that allows us to reduce the four FOC (7), (8), (9) and (10) in one single equation without the endogenous variables \( \phi_1 \) and \( \phi_2 \) explicitly
\[
e_t = \frac{\lambda_2 + \lambda_3 (1 + \beta)}{\beta \lambda_3} e_{t-1} - \frac{\lambda_2}{\beta \lambda_3} e^n - \frac{1}{\beta} e_{t-2} \\
+ \left[ \left( \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \lambda_3} \right) \left( \frac{n_1}{m_1} \right) - \frac{\alpha_1 n_1 + \alpha_2 n_2}{\beta \lambda_3} \right] x_{t-1} \\
- \left( \frac{\alpha_1 m_1 + \alpha_2 m_2}{\lambda_3} \right) \left( \frac{\lambda_4}{\sigma m_1} \right) i_t + \left( \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \lambda_3} \right) \left( \frac{\lambda_4}{\sigma m_1} \right) (1 + \beta) i_{t-1} \\
+ \left( \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \lambda_3} \right) \left( \frac{\lambda_4}{\sigma m_1} \right) i_{t-2}.
\] (18)

### 4.2 A modified Taylor Rule with Interest Rate Smoothing

From (4) one has the following expression rearranged such that \( i_t \) is on the left hand side
\[
i_t = E_t \pi_{t+1} + \frac{E_t x_{t+1}}{\sigma} - \frac{x_t}{\sigma} + \frac{\delta}{\sigma} x_{t-1} + r^n + \frac{\alpha_1}{\sigma} e_t - \frac{\alpha_1}{\sigma} e^n + \frac{\varepsilon_{1t}}{\sigma}
\]
while from (5) we get
\[
E_t \pi_{t+1} = \frac{\pi_t - k x_t - \gamma \pi_{t-1} - \alpha_2 (e_t - e^n) - \varepsilon_{2t}}{\beta}
\]
which inserted in the former equation yields
\[
i_t = E_t \pi_{t+1} + \frac{E_t x_{t+1}}{\sigma} - \frac{x_t}{\sigma} + \frac{\delta}{\sigma} x_{t-1} + r^n + \frac{\alpha_1}{\sigma} e_t - \frac{\alpha_1}{\sigma} e^n + \frac{\varepsilon_{1t}}{\sigma} \\
= r^n - \frac{\alpha_1}{\sigma} e^n + \frac{\alpha_2}{\beta} e^n + \frac{E_t x_{t+1}}{\sigma} - \left( \frac{k}{\beta} + \frac{1}{\sigma} \right) x_t + \frac{\delta}{\sigma} x_{t-1} + \frac{\pi_t}{\beta} - \frac{\gamma}{\beta} \pi_{t-1} \\
+ \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) e_t + \frac{\varepsilon_{1t}}{\sigma} - \frac{\varepsilon_{2t}}{\beta}.
\]
Note that in this last equation we have not made use of the four first order conditions derived in solving the minimization problem. Since equation (18) collapsed all first order conditions in one single equation, where \( e_t \) is a function of the other endogenous variables, we can actually insert equation (18) in this last equation to arrive at

\[
    i_t = r^n - \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) e^n - \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{\lambda_2}{\beta \lambda_3} \right) e^n \\
    + \frac{1}{\sigma} E_t x_{t+1} - \left( k + \frac{1}{\sigma} \right) x_t + \frac{\delta}{\sigma} x_{t-1} + \frac{1}{\beta} \pi_t - \frac{\gamma}{\beta} \pi_{t-1} \\
    + \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \lambda_3} \right) \left( \frac{n_1}{m_1} \right) - \frac{\alpha_1 n_1 + \alpha_2 n_2}{\beta \lambda_3} x_{t-1} \\
    + \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{\lambda_2 + \lambda_3 (1 + \beta)}{\beta \lambda_3} \right) e_{t-1} \\
    - \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{1}{\beta} \right) e_{t-2} \\
    - \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \lambda_3} \right) \left( \frac{\lambda_4}{\sigma m_1} \right) i_t \\
    + \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \lambda_3} \right) \left( \frac{\lambda_4}{\sigma m_1} \right) (1 + \beta) i_{t-1} \\
    + \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \lambda_3} \right) \left( \frac{\lambda_4}{\sigma m_1} \right) i_{t-2} \\
    + \frac{\varepsilon_{1t}}{\sigma} - \frac{\varepsilon_{2t}}{\beta}
\]

which rearranged leaving \( i_t \) on the left hand side gives us finally
\[ i_t = \frac{r^n}{\Gamma} - \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( 1 + \frac{\lambda_1}{\beta \lambda_3} \right) e^n + \frac{1}{\sigma \Gamma} E_t x_{t+1} - \left( \frac{k \sigma + \beta}{\beta \sigma \Gamma} \right) x_t + \left[ \frac{\delta}{\Gamma \sigma} + \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta m_1} \right) \left( \frac{n_1}{\Gamma \lambda_3} - \frac{\alpha_1 n_1 + \alpha_2 n_2}{\Gamma \beta \lambda_3} \right) \right] x_{t-1} + \frac{1}{\beta \Gamma} \pi_t - \frac{\gamma}{\beta \Gamma} \pi_{t-1} + \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{\lambda_2 + \lambda_3 (1 + \beta)}{\Gamma \beta \lambda_3} \right) e_{t-1} - \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{1}{\Gamma \beta} \right) e_{t-2} + \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta m_1} \right) \left( \frac{\lambda_4}{\Gamma \lambda_3 \sigma} \right) (1 + \beta) i_{t-1} + \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta m_1} \right) \left( \frac{\lambda_4}{\Gamma \lambda_3} \right) i_{t-2} + \frac{\varepsilon_{1t}}{\Gamma \sigma} - \frac{\varepsilon_{2t}}{\Gamma \beta} \right] \]

where
\[ \Gamma \equiv 1 + \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta m_1} \right) \left( \frac{\lambda_4}{\sigma \lambda_3} \right) \].

Equation (19) is the optimal monetary rule that comes out of the setup above and which is a modified Taylor rule. This modified Taylor rule has optimally an interest rate smoothing component. Moreover, since lagged inflation rates show up in equation (19) there is presence of persistency of the inflation rate. Furthermore, it has a forward looking expectations component associated with the output gap as well as a lagged value of the output gap that generates also persistency of this variable in the rule. Moreover, the rule also responds to fluctuations of the real exchange rate by incorporating two lags.

For further analysis note that from (13) and (15) we get
\[ \alpha_1 m_1 + \alpha_2 m_2 = \alpha_1 \left( \frac{\beta k}{\beta^2 \gamma [1 - \beta \delta - \beta^{-1}] + \sigma k} \right) + \alpha_2 \left( \frac{\beta [1 - \beta \delta - \beta^{-1}]}{\beta^2 \gamma [1 - \beta \delta - \beta^{-1}] + \sigma k} \right) = \frac{\beta \left( \alpha_1 k + \alpha_2 \left[ 1 - \beta \delta - \beta^{-1} \right] \right)}{\beta^2 \gamma \left[ 1 - \beta \delta - \beta^{-1} \right] + \sigma k} \]

which then implies
\[ \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta m_1} = \frac{\alpha_1}{\beta} - \alpha_2 \left( \frac{1 - \beta + \beta^2 \delta}{\beta^2 k} \right). \]
Hence we can write $\Gamma$ as

$$\Gamma = 1 + \left( \frac{\lambda_4}{\beta \lambda_3} \right) \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \left( \frac{1 - \beta + \beta^2 \delta}{\sigma k} \right) \right). \quad (21)$$

### 4.3 Analyzing the Modified Taylor Rule

It is instructive to study some of the theoretical predictions of our model. Note first that this model can generate the simple Taylor rule for a closed economy with naive expectations on the output gap. To see this note that under naive expectations $E_t x_{t+1} = x_t$, $\alpha_1 = 0$, $\alpha_2 = 0$, $\lambda_3 > 0$, $\lambda_4 > 0$ and $\gamma = 0$ (which together imply $\Gamma = 1$) equation (19) turns out to be

$$i_t = r^n + \left[ \frac{\delta}{\sigma} - \frac{k}{\beta} \right] x_t + \frac{1}{\beta} \pi_t + \frac{\varepsilon_{1t}}{\sigma} - \frac{\varepsilon_{2t}}{\beta}. \quad (22)$$

which is similar to the simple Taylor rule proposed originally by Taylor (1993). As showed above Taylor proposed a rule for the United States in which the coefficient associated with $x$ could be around 0.5 while that associated with $\pi$ could be 1.5. In this case the Taylor principle requires that $\frac{1}{\beta}$ be greater than one for monetary policy to be offsetting when inflation rises which is guaranteed under the assumption $\beta \in (0, 1)$. Actually the values 0.5 and 1.5 could be replicated in this model with the parameter set values defined as: $\delta = \frac{4}{5}$, $\beta = \frac{2}{3}$, $k = \frac{1}{5}$, and $\sigma = 1$. With these parameter values we get $\frac{1}{\beta} = 1.5$ and $\frac{\delta}{\sigma} - \frac{k}{\beta} = 0.5$ the values proposed by Taylor for the United States viewed as a closed economy as in equation (1). Importantly, in equation (22) the optimal response to the output gap is not necessarily positive theoretically since it could be the case that $\frac{\delta}{\sigma} - \frac{k}{\beta} < 0$, even though this does not seem to be empirically the case. Moreover, note that demand shocks, represented by an increase in $\varepsilon_{1t}$, increase optimally $i_t$ while supply shocks, represented by an increase in $\varepsilon_{2t}$, decrease $i_t$.

Let us study now the optimal monetary rule for an open economy ($\alpha_1 \neq 0$ and $\alpha_2 \neq 0$) where a central bank cares about real exchange rate targeting, real exchange rate fluctuations as well as nominal interest rate fluctuations ($\lambda_2 > 0$, $\lambda_3 > 0$, $\lambda_4 > 0$). For this part we specialize equation (19) under two assumptions: $\sigma = \beta$ and $k \sigma = 1 - \beta (1 - \beta \delta) > 0$. These two conditions simplify the equation substantially allowing us to discuss the sign of

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20 From equation (21) we get that $\Gamma$ becomes positive and greater than one under the specific parametric value $k \sigma = 1 - \beta (1 - \beta \delta) > 0$ since now $\Gamma$ turns out to be

$$\Gamma = 1 + \left( \frac{\lambda_4}{\beta \lambda_3} \right) \left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right)^2 \geq 1.$$
most of the terms. Hence equation (19) simplifies to

\[
i_t = \frac{r^n}{\Gamma} - (\alpha_1 - \alpha_2) \left( \frac{1 + \lambda_2}{\Gamma \beta^2 \lambda_3} \right) e^n + \frac{1}{\Gamma \beta} E_t x_{t+1} - \left( \frac{1 + \beta^2 \delta}{\Gamma \beta^2} \right) x_t + \left( \frac{1}{\Gamma \beta} \right) \left[ \delta - \left( \frac{\beta \lambda_1 (1 - \beta \gamma)}{\lambda_3 \beta} \right) \left( 1 - \beta + \beta^2 \delta \right) \left( 1 - \beta \gamma \right) \right] x_{t-1} + \frac{1}{\Gamma \beta} \pi_t - \frac{\gamma}{\Gamma \beta} \pi_{t-1} + (\alpha_1 - \alpha_2) \left( \frac{\lambda_2 + \lambda_3 (1 + \beta)}{\Gamma \beta^2 \lambda_3} \right) e_{t-1} - (\alpha_1 - \alpha_2) \left( \frac{1}{\Gamma \beta^2} \right) e_{t-2} + (\alpha_1 - \alpha_2)^2 \left( \frac{\lambda_4}{\Gamma \beta^2 \lambda_3} \right) (1 + \beta) i_{t-1} + (\alpha_1 - \alpha_2)^2 \left( \frac{\lambda_4}{\Gamma \beta^2 \lambda_3} \right) i_{t-2} + \frac{\varepsilon_{1t}}{\beta \Gamma} - \frac{\varepsilon_{2t}}{\beta \Gamma} + \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \sigma m_1} \left( \frac{\lambda_4}{\Gamma \lambda_3} \right) - \frac{\alpha_1}{\beta} \left( \frac{\alpha_2}{\beta} \right) \left( \frac{\lambda_4}{\Gamma \sigma \lambda_3} \right)
\]

where we have used (13) and (15) as well as

\[
\left( \frac{\alpha_1}{\sigma} - \frac{\alpha_2}{\beta} \right) \left( \frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \sigma m_1} \right) \left( \frac{\lambda_4}{\Gamma \lambda_3} \right) = \left( \frac{\alpha_1}{\beta} - \frac{\alpha_2}{\beta} \right)^2 \left( \frac{\lambda_4}{\Gamma \sigma \lambda_3} \right)
\]

and

\[
\alpha_1 n_1 + \alpha_2 n_2 = \left( \frac{(\alpha_2 - \alpha_1 \beta \gamma)}{\beta \gamma \left[ 1 - \beta \delta - \beta^{-1} \right] + k} \right)
\]

and that

\[
\delta + \left( \frac{\alpha_1 - \alpha_2}{\lambda_3} \right) \left( \left( \frac{\alpha_1}{\beta} - \frac{\alpha_2}{\beta} \right) n_1 - (\alpha_1 n_1 + \alpha_2 n_2) \right) = \delta - \left( \frac{\beta \lambda_1 (1 - \beta \gamma)}{\lambda_3 \beta} \right) \left( 1 - \beta \gamma \right)
\]

Under the two assumptions that we have imposed we get from equation (23) that coefficients associated to \( E_t x_{t+1}, \pi_t, i_{t-1} \) and \( i_{t-2} \) are positive while those associated to \( x_t \) and \( \pi_{t-1} \) are negative. Moreover, coefficients associated to \( x_{t-1}, e_{t-1} \) and \( e_{t-2} \) have an ambiguous sign.

In the short run an optimal response for an inflation rate change \( \pi_t \) is attenuated with respect to an identical closed economy. To see this note that the coefficient associated with \( \pi_t \) in the modified Taylor rule (23) is \( \frac{1}{\Gamma \beta} \) while in the closed economy Taylor rule (22) is \( \frac{1}{\beta} \). Hence, under the two stated assumptions above we have that \( \frac{1}{\Gamma \beta} \leq \frac{1}{\beta} \) given that \( \Gamma \geq 1 \). Moreover, we conclude that interest rate smoothing of the optimal monetary rule is to increase the nominal interest rate during two consecutive periods, given a contemporaneously increase in the interest rate. This is seen in equation (23) since the coefficients on \( i_{t-1} \) and \( i_{t-2} \)
are positive. Interestingly note that the coefficient associated with $i_{t-1}$ in equation (23) is greater in absolute value than the coefficient associated with $i_{t-2}$ since

$$(\alpha_1 - \alpha_2)^2 \left( \frac{\lambda_4}{\Gamma \beta^2 \lambda_3} \right) (1 + \beta) > (\alpha_1 - \alpha_2)^2 \left( \frac{\lambda_4}{\Gamma \beta^2 \lambda_3} \right)$$

given that $\beta > 0$. These conclusions are summarized as propositions.

**Proposition 1** Under $\sigma = \beta$ and $k \sigma = 1 - \beta (1 - \beta \delta)$ the modified Taylor rule in equation (23) is less responsive in the short run to changes in the inflation rate $\pi_t$ compared to an identical closed economy as in equation (22).

**Proposition 2** Moreover, the optimal monetary rule implies that interest rate smoothing has the feature of increasing the nominal interest rate during consecutive periods after a change in the nominal interest rate in period $t$; the increase being greater in period $t+1$ than in $t+2$.

Nonetheless, we need to study the long run effect on the nominal interest rate of changes in $\pi_t$ and $x_t$ given that the modified Taylor rule in equation (23) has interest rate smoothing which is captured by the terms $i_{t-1}$ and $i_{t-2}$. But to do so we need to guarantee the existence of a stationary state of a corresponding dynamical system that subsumes the derived monetary rule. It turns out that we can rewrite the system (4), (5), (16) and (17) recursively as a discrete linear dynamical system in order to show the existence of a stationary state. The following proposition is proven in the appendix.

**Proposition 3** The dynamical system conformed by (4), (5), (16) and (17) has a unique steady state if $\sigma k = 1 - \beta (1 - \beta \delta)$, $\sigma = \beta$, $1 \geq \beta + \gamma$, $\alpha_2 \geq \alpha_1 \geq 0$.

Under the above conditions we can compute the long run effects of a unit increase in $\pi$ and $x$. Consider first the Taylor principle that says that a change in $\pi_t$ must generate a greater increase in the nominal interest rate to offset the inflation surge. This comes down to studying the long run effect of a unit change in $\pi_t$ which is given by

$$LR_\pi = \frac{1 - \gamma}{\Gamma \beta - (\alpha_1 - \alpha_2)^2 \left( \frac{\lambda_4}{\beta \lambda_3} \right) (2 + \beta)}.$$  \hspace{1cm} (24)

If the Taylor principle is to hold we need to verify that the long run effect satisfies $LR_\pi > 1$. Under the two simplifying assumptions in this section and using $\Gamma = 1 + \left( \frac{(\alpha_1 - \alpha_2)^2 \lambda_4}{\beta \lambda_3} \right)$ in (24) yields

$$\beta + \left( \frac{(\alpha_1 - \alpha_2)^2 \lambda_4}{\beta \lambda_3} \right) - \left( \frac{(\alpha_1 - \alpha_2)^2 \lambda_4}{\beta \lambda_3} \right) (2 + \beta) = \frac{1 - \gamma}{\beta - \left( \frac{(1 + \beta)(\alpha_1 - \alpha_2)^2 \lambda_4}{\beta \lambda_3} \right)}.$$
Note that a necessary condition for the Taylor principle to hold is that \( \beta > \frac{(1+\beta)(\alpha_1-\alpha_2)^2\lambda_4}{\beta\lambda_3} \) since only in this case the long run effect could actually be positive given that \( 1 > \gamma > 0 \) by assumption. Nonetheless, the optimal monetary reaction of an open economy to changes in \( \pi \) satisfies the Taylor principle only for values of \( \gamma \) close to zero. To see why this is the case note first that under \( \beta > \frac{(1+\beta)(\alpha_1-\alpha_2)^2\lambda_4}{\beta\lambda_3} \) for \( \gamma = 0 \) the coefficient on the long run effect of a unit increase in \( \pi_t \) is \( \frac{1}{\beta - \frac{(1+\beta)(\alpha_1-\alpha_2)^2\lambda_4}{\beta\lambda_3}} \) which is strictly greater than one satisfying the Taylor Principle given that \( \beta \in (0, 1) \). Now when \( \gamma \) increases towards one the coefficient \( \frac{1}{\beta - \frac{(1+\beta)(\alpha_1-\alpha_2)^2\lambda_4}{\beta\lambda_3}} \) decreases and could eventually become less than one even if \( \alpha_1 = \alpha_2 \). Hence, the Taylor principle is satisfied only for values of \( \gamma \in (0, \tilde{\gamma}) \) where \( \tilde{\gamma} \equiv 1 - \beta + \left( \frac{(1+\beta)(\alpha_1-\alpha_2)^2\lambda_4}{\beta\lambda_3} \right) > 0 \). We summarize this as a proposition.

**Proposition 4** Under \( \sigma = \beta \) and \( k\sigma = 1 - \beta (1 - \beta\delta) \) the modified Taylor rule in equation (23) satisfies the Taylor principle in the long run if \( \beta > \frac{(1+\beta)(\alpha_1-\alpha_2)^2\lambda_4}{\beta\lambda_3} \) and \( \gamma \in (0, \tilde{\gamma}) \).

On the other hand, it is theoretically possible for the optimal rule to violate the Taylor principle in an open economy which would be the case when \( \beta < \frac{(1+\beta)(\alpha_1-\alpha_2)^2\lambda_4}{\beta\lambda_3} \). In this case a unit change effect of \( \pi \) would actually end up reducing the interest rate! This result is more likely to show up the greater the difference between \( \alpha_1 \) and \( \alpha_2 \) for given values of \( \beta \), \( \lambda_3 \) and \( \lambda_4 \).

Consider now the long run effect on the nominal interest rate for a unit increase in the output gap. Assume that expectations are rational such that \( E_t x_{t+1} = x_{t+1} \) and for simplicity assume that \( \alpha_1 = \alpha_2 \) which implies \( \Gamma = 1 \). The long effect according to equation (23) is

\[
LR_x = \frac{1}{\beta} (1 + \delta) - \frac{1}{\beta^2} - \delta
\]

which is not necessarily positive since for values of \( \delta \) close to zero the expression is negative. Here we conclude that the optimal reaction to changes in the output gap for an open economy is not necessarily greater or smaller than the reaction of an identical closed economy. We summarize this also as a proposition.

**Proposition 5** For an open economy the modified Taylor rule in equation (23) does not necessarily generate a greater or smaller response on the nominal interest rate than what is optimal for an otherwise identical closed economy according to equation (22).

Woodford (2003a) has argued that optimal monetary rules can end up having an interest rate smoothing component which is absent in the original Taylor rule. He has argued this for a closed economy i.e. an economy in which the real exchange rate does not have a role
to play. Interestingly in our model we end up having interest rate smoothing only if the economy is actually open i.e. \( \alpha_1 > 0 \) and \( \alpha_2 > 0 \) such that \( \alpha_1 \neq \alpha_2 \) under \( \sigma = \beta \) and \( k \sigma = 1 - \beta (1 - \beta \delta) \). As can be seen in equation (23) when \( \alpha_1 = \alpha_2 = 0 \) we end up without interest rate smoothing. To be sure we are not arguing that in a general model a closed economy cannot have an optimal monetary rule with interest rate smoothing. Actually Woodford (2003a) has shown that interest rate smoothing can show up in the optimal policy of a closed economy even if interest rate changes is not a social objective in itself for a central bank. What we are arguing is simply that in our simple model interestingly we generate interest rate smoothing optimally only for an open economy but not for a closed one.

4.4 Empirical Specification

In this part we tie the theoretical part with an empirical specification. First consider the modified Taylor rule derived above given by equation (19), which does not impose any restriction, under rational expectations \( E_t x_{t+1} = x_{t+1} \) which has as its reduced form

\[
\begin{align*}
  i_t &= \beta_1 + \beta_2 x_{t+1} + \beta_3 x_t + \beta_4 x_{t-1} + \beta_5 \pi_t + \beta_6 \pi_{t-1} + \beta_7 e_{t-1} \\
  &= X_t \beta + u_t
\end{align*}
\]

where the fundamental parameters are combined in the reduced form parameters \( \beta' = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9, \beta_{10}) \), \( X_t = (x_{t+1}, x_t, x_{t-1}, \pi_t, \pi_{t-1}, e_{t-1}, e_{t-2}, i_{t-1}, i_{t-2}) \) and \( u_t = \frac{\varepsilon_{1t} - \varepsilon_{2t}}{\sigma_1^2 - \sigma_2^2} \). Under the above assumptions that \( \varepsilon_{1t}, \varepsilon_{2t} \) are iid normally distributed with mean zero and variances \( \sigma_1^2 \) and \( \sigma_2^2 \) respectively we get that \( u_t \) is iid normally distributed with conditional mean given by \( E(u_t|X_t) = 0 \) and variance given by \( \sigma^2 = Var(u_t|X_t) = \frac{1}{\sigma^2} \left( \frac{\sigma_1^4}{\sigma_2^2} + \frac{\sigma_2^4}{\sigma_1^2} \right) \) for all \( t \) and serially uncorrelated.

Given the review of economic history done above about Banco de la República from 1991 up to 2010 we specify a Markov switching regime model that allows the nominal interest rate to be in two different states, each of which is characterized by a different inflation rate behavior related to the monetary policy strategy. We restrict the state space to two regimes denoted as state 0 and state 1 where the former is labeled as explicit inflation targeting regime (mainly through an interest rate reaction function) while the latter is labeled as non explicit inflation targeting regime (through either monetary aggregates or exchange rate targeting). Denote as \( S_t \in \{0, 1\} \) the state space variable which is discrete and unobserved and which is associated to the state of the economy. This state variable \( S_t \) is governed by the transition probabilities given by

\[
\begin{align*}
  \Pr[S_t = 0|S_{t-1} = 0] &= p; & \Pr[S_t = 1|S_{t-1} = 0] &= 1 - p \\
  \Pr[S_t = 1|S_{t-1} = 1] &= q; & \Pr[S_t = 0|S_{t-1} = 1] &= 1 - q
\end{align*}
\]

22
which gives its non linear Markovian nature. The state variable $S_t$ indexes the parameters of the Taylor rule given in (25) such that

$$
\begin{align*}
i_t &= X_t \beta_{(S_t)} + u_t \\
u_t &\sim N(0, \sigma^2_{(S_t)})
\end{align*}
$$

where

$$
\begin{align*}
\beta_{(S_t)} &= \beta_0 (1 - S_t) + \beta_1 S_t \\
\sigma^2_{(S_t)} &= \sigma^2_0 (1 - S_t) + \sigma^2_1 S_t
\end{align*}
$$

reflects the regime dependency of both the parameters $\beta$ and the variance $\sigma^2$. The empirical model given by (27) was pioneered by Hamilton (1994) and is estimated by a maximum likelihood procedure using Krolzig’s (1997) algorithm.

5 Data

The data was generously provided by Banco de la República. Figures 1 to 4 summarize the four time series in quarterly frequency from 1990 up to 2010 (interest rate, output gap, inflation rate and log of real exchange rate) used in the empirical specification discussed above. All figures display a vertical line that identifies October of 2000 as the date in which explicit inflation targeting was implemented by Banco de la República. As can be seen in the figures the nominal interest rate, output gap and inflation rate seem to have a different behavior after the year 2000 while for the log of the real exchange rate this is not apparent. Importantly, the output gap reflects a huge recession of the Colombian economy which was suffered during 1999 and which coincided shortly after with the adoption of inflation targeting by the central bank of Colombia.
6 Results

Before reporting the two regime Markov switching model for the modified Taylor rule according to (27) we first present a unit root test for the inflation rate. This was done following the lead of Kuzin (2004) and Murray et al. (2008) which suggest that when the Taylor principle is not violated a Taylor rule implies that the inflation rate should be a stationary series while when it does not hold it could become non stationary.

6.1 Markov Switching Unit Root Test

Let $y_t \equiv \Delta \pi_t$ be the first difference of annual quarterly inflation rate for the period 1990 – 2010. Consider the following Markov-switching augmented Dickey-Fuller representation where coefficients and variances are driven by an unobservable state variable $S_t \in \{0, 1\}$

$$
\Delta y_t = \rho(S_t)y_{t-1} + c(S_t) + \theta(S_t)t + \sum_{j=1}^{k} \eta_j(S_t) \Delta y_{t-j} + \nu_t
$$

(28)
where $\nu_t \sim iid(0, \sigma^2_{(S_t)})$. The state variable is governed by a Markov process of order one whose transition probabilities are defined by the following equation

$$
\Pr(S_t = j | S_{t-1} = i, S_{t-2} = h, \ldots, \omega_{t-1}) = \Pr(S_t = j | S_{t-1} = i) = p_{ij}
$$

(29)

where $i, j = 0, 1$ and $\omega_{t-1}$ is the information set up to period $t - 1$. The unit root tests are based on the t-statistic corresponding to the coefficient of $y_{t-1}$ associated with $\rho(S_t) = 0$. In particular, t-statistic is computed as a ratio of the estimated parameter and its standard deviation which is taken from the negative Hessian matrix of the log likelihood function evaluated at the maximum (see Camacho (2010) and Hall et al. (1999)). The steps of the bootstrapping Markov-switching unit root test are the following:

1) Equation (28) is estimated under the null hypothesis and the disturbances are saved and two subsets are formed. The Selection scheme is performed by filtered transition probabilities.

- Subset A1 corresponds to the residuals associated to state 0 or explicit inflation targeting regime.
- Subset A2 corresponds to the residuals associated to state 1 or non explicit inflation targeting regime.

2) Generate a large number $B$ of disturbances with sample size equal to that of the data generating process by bootstrapping the residuals from A1 and A2. It is noted that disturbances are ordered according to the time dimension.

3) Generate a dichotomous state variable using filtered transition probabilities.

4) Generate B realizations of $y_t$ using disturbances obtained through bootstrapping and the estimated parameters where

$$
\Delta y^B_t = C_0 (1 - S_t) + C_1 S_t + \theta_0 (1 - S_t) t + \theta_1 S_t t + \sum_{j=1}^{k} (\eta_{j0} (1 - S_t) \Delta y_{t-j} + \eta_{j1} S_t \Delta y_{t-j}) + \nu^B_{S_t}
$$

5) Equation (28) is estimated for each of $\Delta y^b_t$ for $b = 1, \ldots, B$ and the t-statistic associated to $y^b_{t-1}$ for $b = 1, \ldots, B$ are stored in a vector of size $B \times 1$.

6) The p-value of the unit root test for each state is the percentage of the generated t-ratios that are below the original $t_\rho$.

To test whether the inflation is generated by a unit root Markov-switching processes we apply an ADF Markov-switching unit root test. First equation (28) is estimated for both regimes simultaneously and which is reported in Table 1.
Second we calculate the p-value associated to the t-statistic of the coefficient associated with $\delta_y_1$ through a bootstrapping procedure for each of the regimes as described above with 1000 replications. The result is reported in Table 2 which shows that the null is rejected for the presence of a unit root in regime 0 while it is not rejected in regime 1. As we shall show below this result is consistent with a monetary policy that satisfies the Taylor principle as in regime 0 generating and inflation rate that is a stationary series. In contrast, in regime 1 the Taylor principle is not satisfied by the monetary policy of the central bank and therefore the inflation rate is non stationary or presents a unit root.

### Table 2: Unit Root Bootstrapping Test

<table>
<thead>
<tr>
<th>Regime</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 0:</td>
<td>0.020</td>
</tr>
<tr>
<td>Regime 1:</td>
<td>0.972</td>
</tr>
</tbody>
</table>

#### 6.2 Markov Switching for the modified Taylor Rule

A first order two state Markov switching model was estimated for Colombia with quarterly data of the interest rate and its explanatory variables for the period 1990 – 2010. Table 3 reports the maximum likelihood estimates of the reduced form parameters of the modified Taylor Rule using specification (27) where again 0 denotes a regime labeled as explicit inflation targeting and 1 denotes a regime labeled without explicit inflation targeting. The table reveals different signs for some of the coefficients across regimes. Take for instance the coefficient on the forward term of the output gap ($x_{t+1}$) which is negative in regime 1 while positive in regime 0, both being statistically significant, the first at the 1% while the second at the 10% level. In contrast, the coefficient on the lag term of the output gap ($x_t$) is positive in regime 1 while negative in regime 0 but both are not even statistically significant at the 10%. Similarly, the coefficient on the contemporaneous inflation rate ($\pi$) is positive.
in regime 1 while negative in regime 0 but only the latter is statistically significant at the 10%. The lag of the inflation rate ($\pi_{-1}$) is positive on both regimes but only statistically significant in regime 0. Furthermore, the coefficients associated to the lags on the interest rate ($i_{-1}$ and $i_{-2}$) also switch signs across regimes. The first lag is always statistically significant at the 1% while the second lag is only significant at the 1% in regime 1 when there is no explicit inflation targeting. Furthermore, the exchange rate lags are not statistically significant at any reasonable level in any regime which suggests that the interest rate is not sensitive in any regime to changes in this variable.

The magnitudes of the coefficients also contrast from one regime to the other. Note that the coefficients on the output gap are in magnitude much higher in regime 1 than in regime 0. Actually the long run effect of a unit increase in the output gap in period $t$ is reported in Table 3 below as the coefficient on $LR_x$. As seen the magnitude is 3.263 for this coefficient in regime 1 while only being 0.282 in regime 0. These point estimates suggest that the central bank of Colombia targeted aggressively the fluctuations on the output gap in regime 1 while not in regime 0.

On the other hand, it is striking that the coefficients on the inflation rate in magnitude are quite small in regime 1 while being much higher in regime 0. Actually, as the table reports, the long run effect of a unit increase in the inflation rate in period $t$, denoted $LR_\pi$, is only 0.307 in regime 1 while being 1.120 in regime 0, which is almost a four fold increase. More importantly, these point estimates show evidence that the Taylor principle was only

<table>
<thead>
<tr>
<th>Dep. Vble: $i$</th>
<th>Regime without Explicit Inflation Targeting (1)</th>
<th>Regime with Explicit Inflation Targeting (0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{+1}$</td>
<td>$-8.42$ ($2.29$)</td>
<td>$-3.68$</td>
</tr>
<tr>
<td>$x$</td>
<td>$8.37$ ($4.13$)</td>
<td>$2.02$</td>
</tr>
<tr>
<td>$x_{-1}$</td>
<td>$3.10$ ($3.31$)</td>
<td>$0.93$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$0.09$ ($0.91$)</td>
<td>$0.11$</td>
</tr>
<tr>
<td>$\pi_{-1}$</td>
<td>$0.18$ ($0.87$)</td>
<td>$0.21$</td>
</tr>
<tr>
<td>$e_{-1}$</td>
<td>$0.24$ ($0.31$)</td>
<td>$0.78$</td>
</tr>
<tr>
<td>$e_{-2}$</td>
<td>$-0.30$ ($0.27$)</td>
<td>$-1.11$</td>
</tr>
<tr>
<td>$i_{-1}$</td>
<td>$-0.41$ ($0.16$)</td>
<td>$-2.47$</td>
</tr>
<tr>
<td>$i_{-2}$</td>
<td>$0.47$ ($0.18$)</td>
<td>$2.66$</td>
</tr>
<tr>
<td>intercept</td>
<td>$0.48$ ($1.06$)</td>
<td>$0.45$</td>
</tr>
<tr>
<td>std error</td>
<td>$0.0523$</td>
<td></td>
</tr>
<tr>
<td>$LR_\pi$</td>
<td>$0.307$</td>
<td></td>
</tr>
<tr>
<td>$LR_x$</td>
<td>$3.263$</td>
<td></td>
</tr>
</tbody>
</table>
satisfied in regime 0 but not in regime 1 which is consistent with a unit root for the interest rate in regime 1 while stationary in regime 0.

Table 4 reports the transition matrix between regimes which shows high persistency in regimes. As seen, the probability of switching from regime 1 non explicit inflation targeting state to the inflation target regime 0 is equal to 0.1483, while the probability of switching from the inflation targeting regime 0 to the non explicit inflation targeting state 1 is 0.0642. This shows that once the inflation targeting regime state is reached it is quite unlikely to switch from it

<table>
<thead>
<tr>
<th>Table 4: Transition Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Regime 0</td>
</tr>
<tr>
<td>Regime 0</td>
</tr>
<tr>
<td>Regime 1</td>
</tr>
</tbody>
</table>

Given the estimated transition probabilities, the average length of each state can be calculated using the following equations.

\[
\sum_{i=1}^{\infty} ixq^{(i-1)} (1 - q) ; \quad \sum_{i=1}^{\infty} ixp^{(i-1)} (1 - p)
\]

The first equation in (30) give us the expected duration of state 1 and the second equation the expected duration of state 0. Table 5 shows the properties of each regime. The average length of being in state 0 is 15.58 quarters while for state 1 is only 6.74 quarters.

<table>
<thead>
<tr>
<th>Table 5: Regime Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Regime</td>
</tr>
<tr>
<td>Regime 0</td>
</tr>
<tr>
<td>Regime 1</td>
</tr>
</tbody>
</table>

Figure 5 shows the evolution for each quarter from 1990 to 2010 of the filter and smoothed probabilities of regime 0 while figure 6 of regime 1. As figure 5 reveals regime 0 of explicit inflation targeting is absolutely prevalent after the year 2000 which coincides approximately with the announcement of Banco de la República of implementing explicit inflation targeting. The figure also reveals that regime 0 became prevalent somewhat before the year 2000 which suggests that the central bank adopted a policy consistent with the Taylor principle even before the announcement of adopting explicit inflation targeting in October of 2000. However, the monetary policy followed by the central bank of Colombia between 1990 and 2000 also had a behavior in some quarters consistent with regime 0. How can we explain this? One reason that could explain this result is that the Constitutional mandate of 1991 required from the central bank the announcement to the public of a quantitative target of inflation for each year and the central bank started to implement some elements of an inflation targeting
strategy like models to forecast inflation (Hernández and Tolosa (2001) and Gómez et al. (2002)). Figure 6 shows the evolution of the filter and smoothed probabilities of regime 1. As seen this regime did not appear after the year 1999.

![Regime 0](image)

**Figure 5**

These results shows why we have labeled regime 0 as explicit inflation targeting regime since it became absolutely prevalent after 1999 according to figure 5. Hence, our previous results in table 1 show that during explicit inflation targeting, from 2000 onwards up to 2010, the Taylor principle was followed and made the inflation rate a stationary series as the results reported above. Moreover, regime 1 did not follow the Taylor principle and therefore the inflation rate was non stationary also consistent with our results above for the unit root test.\(^{21}\) Furthermore, these results contrast starkly with Bernal (2002) since she finds for the entire period.

---

\(^{21}\)Echavarría, López and Misas (2010) study the annualized inflation rate for the same period (1990-2010) and find significant changes in the mean and variance of the time series for the period 1990:1-2000:1 in contrast with the period 2000:2 - 2010:6. Nonetheless, they fail to find unit root behavior for the whole period even though they do find that the sum of autorregressive coefficients for the inflation series, an indicator of persistency, did not drop in Colombia with the adoption of inflation targeting by the central bank. On the other hand, Echavarría, Rodriguez and Rojas (2010) find a similar result as ours since they
period 1991 to 1999 a coefficient less than one on the output gap in a simple Taylor rule with interest rate smoothing while we find a coefficient less than one for the same period in which regime 1 was prevalent. Also Bernal (2002) finds for the period 1991 to 1999 a coefficient greater than one on the inflation rate which goes against our results that suggest that in this period regime 1 was prevalent and accordingly the Taylor principle was violated.

**Figure 6**

### 7 Conclusions

This article has reconstructed the history of monetary policy of the central bank of Colombia in the period 1990 to 2010 in which explicit inflation targeting was adopted by October of year 2000. We develop a theoretical modified Taylor rule with interest rate smoothing for an open and small economy and accordingly estimate a two regime Markov switching model. We obtain low persistence of the quarterly annualized inflation rate from 1999 to 2010 (which they attribute to the explicit adoption of inflation targeting in October of 2000 by Banco de la República) as well as finding high persistence of this time-series for the periods 1979-1989 and 1989 to 1999.
find that one regime had explicit inflation targeting (from the year 2000 up to 2010) in which
the inflation rate is a stationary series given that central bank enforced a monetary policy
that satisfied the Taylor principle. The prevalent regime before the year 2000 did not satisfy
the Taylor principle allowing a unit root behavior of the inflation rate. Moreover we find that
the central bank reacted aggressively during the latter regime (predominantly during the
nineteen nineties) to output fluctuations while having an accommodating behavior for this
variable during explicit inflation targeting from 2000 onwards.

For the future we want to test for further features of our model and also develop a struc-
tural VAR model based on our theoretical framework. We believe this is a quite promising
area for future research in Taylor rules for small and open economies as the one in which we
have probed our model.

Appendix

Linear Dynamical System

We can rewrite the system (4), (5), (16) and (17) recursively as a discrete linear dynamical
system in order to show the existence of a stationary state. Consider first equation (16)
which can be forward one period and rearranged to yield

\[ E_{t}i_{t+1} = (\pi_{t} - \bar{\pi}) \frac{\sigma m_{1}}{\beta \lambda_{4}} + \frac{(1 + \beta)}{\beta} i_{t} + \frac{1}{\beta} i_{t-1} + \frac{n_{1} \sigma}{\beta \lambda_{4}} x_{t} \]  

(31)

while equation (32) can also be forward one period to get

\[ E_{t}e_{t+1} = \left( \frac{\lambda_{2} + \lambda_{3} (1 + \beta)}{\beta \lambda_{3}} \right) e_{t} - \left( \frac{\lambda_{2}}{\beta \lambda_{3}} \right) e^{n} - \frac{1}{\beta} e_{t-1} - \left( \frac{\alpha_{1} m_{1} + \alpha_{2} m_{2}}{\beta \lambda_{3}} \right) (\pi_{t} - \bar{\pi}) - \left( \frac{\alpha_{1} n_{1} + \alpha_{2} n_{2}}{\beta \lambda_{3}} \right) x_{t}. \]

(32)

Now equations (4) and (5) can be rewritten as

\[ E_{t}x_{t+1} = \sigma i_{t} + x_{t} - \delta x_{t-1} - \sigma E_{t} \pi_{t+1} - \alpha_{1} e_{t} - \sigma r^{n} + \alpha_{1} e^{n} - \varepsilon_{1t} \]  

(33)

and

\[ E_{t} \pi_{t+1} = \frac{\pi_{t} - \gamma \pi_{t-1} - k x_{t} - \alpha_{2} e_{t} + \alpha_{2} e^{n} - \varepsilon_{2t}}{\beta}. \]

(34)
Replace (34) in (33) to get

\[ E_t x_{t+1} = \left(1 + \frac{\sigma k}{\beta}\right) x_t - \left(\frac{\sigma}{\beta}\right) \pi_t + \sigma i_t - \left(\alpha_1 - \frac{\alpha_2 \sigma}{\beta}\right) e_t \]

\[ -\delta x_{t-1} + \left(\frac{\sigma \gamma}{\beta}\right) \pi_{t-1} + \left(\alpha_1 - \frac{\alpha_2 \sigma}{\beta}\right) e^n - \sigma r^n \]

\[ -\varepsilon_{1t} + \left(\frac{\sigma}{\beta}\right) \varepsilon_{2t} \]

Under \( \varepsilon_{1t} = \varepsilon_{2t} = 0 \) for all \( t \) in a steady state we can write equations (31), (32), (34) and (35) as a second order non-homogenous linear system

\[ Z_{t+2} = A Z_{t+1} + B Z_t + C \]

where \( Z_{t+1} = \begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ E_t i_{t+1} \\ E_t \varepsilon_{t+1} \end{bmatrix} \), \( A = \begin{bmatrix} 1 + \frac{\sigma k}{\beta} & -\frac{\sigma}{\beta} & \sigma & -\left(\alpha_1 - \frac{\alpha_2 \sigma}{\beta}\right) \\ -\frac{k}{\beta} & \frac{1}{\beta} & 0 & -\frac{\alpha_2}{\beta} \\ -\frac{\alpha_1 \sigma}{\beta \lambda_4} & -\frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \lambda_4} & 0 & \lambda_2 + \lambda_3 (1+\beta) \\ -\frac{\alpha_1 m_1}{\beta \lambda_3} & -\frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \lambda_3} & 0 & \lambda_2 + \lambda_3 (1+\beta) \end{bmatrix} \), \( B = \begin{bmatrix} -\delta & \frac{\sigma \gamma}{\beta} & 0 & 0 \\ 0 & -\frac{\gamma}{\beta} & 0 & 0 \\ 0 & 0 & \frac{1}{\beta} & 0 \\ 0 & 0 & 0 & -\frac{1}{\beta} \end{bmatrix} \) and \( C = \begin{bmatrix} \left(\alpha_1 - \frac{\alpha_2 \sigma}{\beta}\right) e^n - \sigma r^n \\ \alpha_2 e^n \\ -\bar{n} \frac{\alpha_1 m_1}{\beta \lambda_4} \\ \left(\frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \lambda_3}\right) \bar{n} - \left(\frac{\lambda_2}{\beta \lambda_3}\right) e^n \end{bmatrix} \).

A convenient way to analyze this system is to transform it into a first order non-homogenous linear system. To do so define \( Z_{t+1} = Y_t \) so that \( Z_{t+2} = Y_{t+1} \) and rewrite (36) as

\[ \begin{bmatrix} Y_{t+1} \\ Z_{t+1} \end{bmatrix} = \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} \begin{bmatrix} Y_t \\ Z_t \end{bmatrix} + \begin{bmatrix} C \\ 0 \end{bmatrix} \]

or equivalently as

\[ W_{t+1} = MW_t + N \]

where \( W_t = \begin{bmatrix} Y_t \\ Z_t \end{bmatrix} \), \( M = \begin{bmatrix} A & B \\ I_4 & 0 \end{bmatrix} \) and \( N = \begin{bmatrix} C \\ 0 \end{bmatrix} \). Note that \( W \) is a 8x1 vector as well as \( N \) while \( M \) is an 8x8 matrix.

**Existence and Uniqueness of a Steady State**

**Proposition 6** The dynamical system (37) has a unique steady state if \( \sigma k = 1 - \beta (1 - \beta \delta) \), \( \sigma = \beta, 1 \geq \beta + \gamma, \alpha_2 \geq \alpha_1 \geq 0 \).
To prove this result note that in steady state one should have \( W_t = \tilde{W} \) for all \( t \) in (37). Hence to prove existence and uniqueness of a steady state one should find the conditions that guarantees that \( [I - M]^{-1} \) exists so that one can compute
\[
\tilde{W} = [I - M]^{-1} \tilde{N}.
\]
Hence a necessary condition is that \( \text{det} [I - M] \neq 0 \). To get the sufficient conditions under which this determinant is non zero note that
\[
I - M = \begin{bmatrix} I_4 & 0 \\ 0 & I_4 \end{bmatrix} - \begin{bmatrix} A & B \\ I_4 & 0 \end{bmatrix} = \begin{bmatrix} I_4 - A & -B \\ -I_4 & I_4 \end{bmatrix}.
\]
By the formula for the determinant of a partition matrix\(^{22}\) one has
\[
|I - M| = |I_4| \cdot |I_4 - A - B| = |I_4 - A - B|.
\]
Therefore consider
\[
I_4 - A - B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 + \frac{\sigma k}{\beta} & -\frac{\sigma}{\beta} & \sigma & -\left(\alpha_1 - \frac{\alpha_2 \sigma}{\beta}\right) \\ -\frac{\sigma}{\beta} & \frac{1}{\beta} & 0 & -\frac{\sigma_2}{\beta} \\ -\frac{\alpha_1 \sigma}{\beta \lambda_4} & \frac{\sigma m_1}{\beta \lambda_4} & \frac{(1+\nu)}{\beta} & 0 \\ -\frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \lambda_3} & -\frac{\alpha_1 m_1 + \alpha_2 m_2}{\beta \lambda_3} & 0 & \frac{\lambda_2 + \lambda_3 (1+\nu)}{\beta \lambda_3} \end{bmatrix}.
\]
Let a given matrix be partitioned in a 2x2 blocked matrix
\[
A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}.
\]
The determinant of \( A \) is given by
\[
|A| = |A_{22}| \cdot |A_{11} - A_{12} A_{22}^{-1} A_{21}|.
\]

\(^{22}\)
column of the matrix which reduces to

\[
|I_A - B| = -\sigma \begin{vmatrix}
\frac{k}{\lambda_3} & 1 - (1-\gamma) & \frac{\alpha_2}{\lambda} \\
-\frac{n_1\sigma}{\lambda_3} & \frac{-\sigma\lambda}{\lambda_3} & \frac{\alpha_2}{\lambda} \\
\frac{\alpha_1m_1 + \alpha_2m_2}{\lambda_3} & \frac{\alpha_1m_1 + \alpha_2m_2}{\lambda_3} & -\lambda_2 \\
\end{vmatrix} = \frac{-2}{\lambda_3} \begin{vmatrix}
\frac{-\sigma k}{\beta} & \frac{\sigma(1-\gamma)}{\beta} & \frac{\alpha_1 - \alpha_2\sigma}{\beta} \\
\frac{k}{\beta} & \frac{1}{\lambda_3} & \frac{1}{\lambda_3} \\
\frac{\alpha_1m_1 + \alpha_2m_2}{\lambda_3} & 1 - (1-\gamma) & \frac{\sigma(1-\gamma)}{\beta} \\
\end{vmatrix}
\]

Equivalently the determinant of \(|I_A - B|\) is

\[
|I_A - B| = \left(\frac{-\sigma^2\alpha_2}{\lambda_3^2 4\lambda_3}\right) [n_2m_1 - n_1m_2] - \left(\frac{\lambda_2\alpha_2}{\lambda_3^2 4\lambda_3}\right) [km_1 - n_1(1-\beta-\gamma)]
\]

Under \(\sigma k = 1 - \beta (1-\beta \delta)\), \(\sigma = \beta\) and the definitions of \(m_1, m_2, n_1, n_2\) we have that

\[
n_2m_1 - n_1m_2 = \frac{\lambda_2\alpha_2}{\lambda_3^2 4\lambda_3} [km_1 - n_1(1-\beta-\gamma)]
\]

which is positive as well as

\[
km_1 - n_1(1-\beta-\gamma) = \frac{\beta k^2}{\lambda_1 (1-\beta\gamma)}
\]
given that $1 \geq \beta + \gamma$. Moreover note that

$$\lambda_2 (1 - \beta - \gamma) - \alpha_2 (\alpha_1 m_1 + \alpha_2 m_2) = \lambda_2 (1 - \beta - \gamma) - \alpha_2 \left( \frac{\beta (\alpha_1 k + \alpha_2 [1 - \beta \delta - \beta^{-1}])}{\sigma k - \beta \gamma [1 - \beta (1 - \beta \delta)]} \right)$$

$$= \lambda_2 (1 - \beta - \gamma) + \alpha_2 \left( \frac{\alpha_2 - \alpha_1}{1 - \beta \gamma} \right)$$

is positive under $1 \geq \beta + \gamma$ and $\alpha_2 \geq \alpha_1 \geq 0$. Furthermore note that

$$\lambda_2 \sigma (1 - \gamma) + (\alpha_1 \beta - \alpha_2 \sigma) (\alpha_1 m_1 + \alpha_2 m_2) = \lambda_2 \sigma (1 - \gamma) + \frac{\beta (\alpha_1 - \alpha_2)^2}{(1 - \beta \gamma)}$$

which is always positive. Consider now

$$\frac{\alpha_1 n_1 + \alpha_2 n_2}{\beta} = \frac{\lambda_1 (\alpha_2 - \alpha_1 \beta \gamma)}{\beta k (1 - \beta \gamma)}$$

which is positive under $\alpha_2 \geq \alpha_1 \geq 0$. Finally consider

$$\alpha_2 \sigma (1 - \gamma) + (1 - \beta - \gamma) (\alpha_1 \beta - \alpha_2 \sigma) = \beta [\alpha_1 (1 - \gamma) + \beta (\alpha_2 - \alpha_1)]$$

which is also positive under $\alpha_2 \geq \alpha_1 \geq 0$. Taking into account all the terms developed above and replacing them in (38) under $\sigma k = 1 - \beta (1 - \beta \delta)$ which implies $\delta - \frac{\sigma k}{\beta} = \frac{\beta \delta (1 - \beta) + (1 + \beta)}{\beta}$ yields

$$|I_4 - A - B| = - \left( \frac{\alpha_2^2}{\lambda_4 \lambda_3} \right) \left[ \frac{\lambda_1}{k (1 - \beta \gamma)} \right] - \left( \frac{\lambda_2}{\lambda_4 \lambda_3} \right) \left[ \frac{k^2 + \beta \gamma \lambda_1 (1 - \beta - \gamma)}{k (1 - \beta \gamma)} \right]$$

$$- \left( \frac{2}{\beta^3 \lambda_3} \right) \left[ \beta \delta (1 - \beta) + (1 + \beta) \right] \left[ \lambda_2 (1 - \beta - \gamma) + \alpha_2 \left( \frac{\alpha_2 - \alpha_1}{1 - \beta \gamma} \right) \right]$$

$$- 2 \left( \frac{k}{\beta^2 \lambda_3} \right) \left[ \lambda_2 (1 - \gamma) + \frac{(\alpha_1 - \alpha_2)^2}{1 - \beta \gamma} \right]$$

$$- 2 \left( \frac{1}{\beta^2 \lambda_3} \right) \left( \frac{\lambda_1 (\alpha_2 - \alpha_1 \beta \gamma)}{k (1 - \beta \gamma)} \right) \left[ \alpha_1 (1 - \beta - \gamma) + \alpha_2 \beta \right].$$

which has a strictly negative sign. Hence a unique steady state exists.
References


