Poverty Traps, Economic Inequality and Delinquent Incentives

Andrés Salazar
Departamento Nacional de Planeación

Edgar Villa
Departamento de Economía
Pontificia Universidad Javeriana

Abstract

This paper explores theoretical linkages between poverty traps, economic inequality and delinquency in a two sector overlapping generations model under perfect competition in which barriers to skilled educational attainment and delinquent incentives interact. We find that the existence of a poverty trap under high economic inequality and costly indivisible human capital investments generate persistent delinquency. We study shocks that increase skilled wages or reduce assets for the unskilled and find that these temporal shocks produce an outburst of delinquency in the short run that die out later on. If the shock is permanent then delinquency increases permanently in the long run. Furthermore, we find that when law enforcement policies increase deterrence and incapacitation permanently delinquency diminishes en the long run but is accompanied by an increase in wealth inequality. We also find that subsidies for human capital investments can have an ambiguous effect on delinquency in the long run.

JEL code: I30, J31, K42, O11, O17
Key words: Poverty Traps, Inequality, Delinquency, Human Capital
Trampas de Pobreza, Desigualdad Económica e Incentivos para la Delincuencia

Andrés Salazar
Departamento Nacional de Planeación

Edgar Villa
Departamento de Economía
Pontificia Universidad Javeriana

Resumen

Este artículo explora la conexión teórica entre trampas de pobreza, desigualdad económica y delincuencia en un modelo de dos sectores de generaciones traslapadas bajo competencia perfecta en la que barreras al acceso a educación e incentivos para la delincuencia interactúan. Encontramos que la existencia de una trampa de pobreza bajo una alta desigualdad económica e indivisibilidad costosa de la inversión en capital humano generan delincuencia persistente. Estudiamos choques que incrementan el salario de trabajadores educados o que reduzcan los activos para los trabajadores no educados los cuales si son temporales generan aumentos en delincuencia en el corto plazo que luego tienden a diluirse en el largo plazo. Si el choque es permanente entonces se produce un aumento en la delincuencia en el largo plazo. Encontramos además que políticas que aumenten la seguridad permanentemente disminuyen la delincuencia en el largo plazo pero acompañados de un aumento en la desigualdad económica. Más aún encontramos que subsidios a inversiones en capital humano pueden tener un efecto ambiguo sobre el nivel de delincuencia en el largo plazo.

Código JEL: I30, J31, K42, O11, O17
Palabras Clave: Trampas de Pobreza, Desigualdad, Delincuencia, Capital Humano
"A person in imminent danger [who] cannot be helped in any other way........may legitimately supply his own wants out of another’s property" (Thomas Aquinas, ed. Gilby (1975))

Introduction

Poverty has been persistent in the history of human economic development. As Aquinas suggests in the citation given above poverty has been associated with delinquency since ancient time, and even the Church justified it to be legitimate in those days under extreme lack of economic opportunities. From a modern economics viewpoint, in which property rights have been privileged, it seems unlikely to justify any type of delinquency even in the face of poverty since for some scholars economic prosperity through investments is the product of a provision of security against delinquency. In this view higher delinquency should cause economic downturn. Nonetheless, delinquency is an economic choice for individuals and presumably, the lack of economic opportunities can generate higher delinquency. Hence, causality between delinquency and economic prosperity can be subject to simultaneity. If so social policies that subsidize education and provide health insurance can end up alleviating poverty which in turn can have an effect in lowering delinquency. This view suggests a carrot type of policy to induce individuals not to choose a delinquent life. On the other hand, since the pioneering work of Becker (1968) and Ehrlich (1973), the economics of crime literature has argued that in order to lower delinquency it is necessary to extend and increase law enforcement policies to deter directly individuals from choosing delinquent activities or to incapacitate them. This view suggests a stick type of policy that punishes individuals that choose a delinquent life. Both views suggest that we should understand the economic incentives for an individual to choose delinquency and how these interact with poverty, economic inequality and law enforcement punishments.

Poverty traps have been studied recently\(^1\) and it is believed that persistent poverty can lower economic growth and increase economic inequality. If delinquent incentives come with poverty then it seems important to understand the connection between poverty traps and delinquency as well. Moreover, in the economics of education literature there is evidence that human capital accumulation can weaken delinquent

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\(^0\) Contact email: e.villa@javeriana.edu.co. We thank the comments of the participants of the Seminars of Pontificia Universidad Javeriana, Universidad Autónoma de Bucaramanga y Universidad de los Andes. We also thank the participants of the seminar at Banco de la República and Departamento Nacional de Planeación. Naturally all remaining errors are ours.

\(^1\) See Azariadis and Stachurski (2005) for a theoretical overview.
incentives (Lochner (2004, 2010), Lochner-Moretti (2001)). In particular, this literature argues that educational attainment is causally related to higher returns in the labor market as well as positive externalities at the social level which suggests that policies that enhance education opportunities for riskier segments of the population have a positive externality that lowers delinquent incentives. All this suggests that we should understand the incentives for an individual to choose a delinquent life in environments where there exists both poverty traps, high economic inequality and barriers to acquire human capital.

This paper builds an overlapping generations model similar to Galor-Zeira (1993) under perfect competition to study the theoretical linkages between poverty traps, economic inequality and human capital attainment. It builds on a dual economy in which delinquents come out of poverty and become parasites that prey on legal workers. It finds that for given levels of law enforcement measures delinquency is persistent in the long run if there is a poverty trap and the economy starts out with enough wealth inequality and a large fraction of unskilled workers that lowers sufficiently the unskilled wage relative to costly indivisible human capital investments. We then study comparative dynamics when parameters are taken to shift temporarily or permanently in the model. We find that policies that increase law enforcement deterrence generate higher inequality in the long run given the existence of a poverty trap.

This paper is organized in five parts. The first part reviews a strand of literature that links both delinquency to economic inequality and poverty while also reviewing another strand of literature that links education attainment and delinquency. The second part builds up the formal model which explores the theoretical linkages between poverty traps, economic inequality and delinquent incentives. The third part explores comparative dynamics with respect to parameter changes. The fourth part concludes.

1 Literature review

The modern literature on the economics of crime, based on Gary Becker’s seminal (1968) article, has focused on the effect of deterrence and incapacitation on criminal behavior. This tradition understands delinquency as a result of individual rational choice where benefits of illegal activities outweigh the costs (punishment in case of apprehension and conviction) as well as their current set of opportunities. As a consequence, deterrence theory research has been predominantly concerned with the isolated effects of the severity and certainty of sanctions on illegal behavior, which has been an argument to extend and increase law enforcement policies in order to reduce
delinquent incentives. However, economic and social literature argue that delinquency is not likely to be eliminated completely only through law enforcement policies since delinquency is an individual choice in the presence of barriers to enter legal sectors that yield economic opportunities (Eide 1997). Specifically, both poverty and high economic inequality are social conditions that induce illegal behaviors due the lack of other legal ways to acquire income and build assets. Still human capital accumulation can have a role to play in reducing the delinquent incentives. We now turn to these two relations.

1.1 Poverty, inequality and delinquency

There is a recent literature in economics that studies institutional changes in a dynamic evolutionary framework where inequality and poverty traps emerge with endogenous inefficient institutional arrangements (Bowles 2006). Other theoretical contributions argue that parasitic enterprises can feed on productive businesses (Mehlum, Moene and Torvik (2003, 2006)) where the fraction of parasitic enterprises is determined endogenously depending on the institutional arrangements in which an economy operates, namely depending crucially on legal versus the illegal opportunities they face. This literature understands poverty traps as dreadful equilibria in a multiple equilibrium environments where inefficient or perverse institutions sustain them and could become persistent in time.

According to Kelly (2000) the link between inequality and crime has been studied by three main theories of crime: economic theory of crime, social disorganization theory, and strain theory. In the economics crime literature, it has been argued that economic differences have been a necessary condition to keep the incentives to commit felonies, hence, property crimes may partly be the consequence of excessive economic inequality (Bourguignon (1999), Fender (1999)). Others have considered the effect of inequality on crime, for example, Ehrlich (1973) uses the fraction of the population in an area earning less than half the median income as a proxy for inequality, and shows that the decision to participate in criminal activities involving material gains is positively associated with income inequality. Witte and Tauchen (1994) examine the impact of earnings on criminal participation and Kelly (2000, pag. 537), using FBI Uniform Crime Reports in US, concludes that "the impact of inequality on violent crime is large, even after controlling for the effects of poverty, race, and family composition". Moreover, some other authors have found evidence of a positive association between income inequality and crime rates using cross country data. For example, Krohn (1976), Soares (2004) and Fajnzylber et al (2002) show that countries with more unequal income distribution tend to have higher crime rates.
than those with more equal patterns of income distribution, for different samples of countries. Another study finds that a one-point rise in a country’s Gini coefficient is associated with nearly a one-point increase in its homicide rate (UN Global Report on Crime and Justice (1999) quoted in Buvinic and Morrison (2000)). Naturally this evidence should not be interpreted causally but only as associations between certain aggregate variables.

The social disorganization theory emphasizes that the existence of several factors such as poverty, family stability, residential mobility and ethnic heterogeneity push some members of communities to illegal activities and weakens the social control of this behavior (Shaw and McKay (1942)). This theory conjectures that income inequality causes delinquency in an indirect way due to the fact that inequality is related with poverty and this factor induces more likely individuals to commit illegal acts.

Finally, strain theory based on Merton’s (1938) work developed the idea of *anomie*, as the lack of social norms or the failure of a social structure to provide mechanisms and pathways necessary for people to achieve their goals, generating deviant behaviors such as crime. In this theory individual alienation can arise from income inequality, and are also related with other measures of deprivation such as poverty and unemployment. This idea is related with the argument that criminality is based on an individual process that consists of an assessment of economic incentives and social norms (Weibull and Villa (2005)).

However, the relationship between income inequality and delinquency and violence is not completely straightforward. Some countries have decreasing income inequality accompanied by an increase in violence (measured in homicide rates) such as Brazil and Venezuela, or a decrease in homicide rates accompanied by an increase in income inequality such as Costa Rica and Mexico (Morrison, Buvinic, and Shifter (2003)). Moreover, for a specific sample in the U.S., income inequality has no significant effects on property crimes such as robbery, burglary and vehicle theft (Allen (1996)).

### 1.2 Education and delinquent incentives

The economics of education literature has found evidence that human capital accumulation can discourage illegal activities. For example, Freeman (1996) shows that educational attainment is a preventive policy for crime and finds an inverse relationship between these two variables. Tauchen, et al (1994) studied a sample of men who attended school relative to those who did not and found a negative relationship between the act of studying or working with the probability of committing criminal

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2To see a complete summary of this evidence, see Soares (2004).
acts. They argue that studying and working are associated with greater participation in legal activities and therefore decrease the incentives to commit illegal acts. Lochner and Moretti (2001) also show that there is an inverse relationship between school attainment and crime rates. They find that youths that finish high school are more likely not to enter in delinquent activities. Moreover, they argue that education has a positive externality in reducing crime (Lochner (2004, 2010)).

2 The model

2.1 Legal and Illegal Sectors

Consider a small open economy that produces an homogenous good that can be used for consumption and investment. The good can be produced by two technologies, one uses skilled labor and capital and the other one uses unskilled labor and land. These define a two legal sector economy that demands labor from potential workers. Nonetheless, some potential workers could choose to become delinquents and enter an illegal sector with the explicit purpose of acquiring the consumption good by targeting workers of the legal sector. Formally, production in the legal skilled labor sector is described by $Y^s_t = F(K^s_t, L^s_t)$ where $Y^s_t$ is output, $K^s_t$ is capital and $L^s_t$ is skilled labor, while $F$ is a concave production function with constant returns to scale. It is assumed that investment in human capital and in physical capital is made one period in advance and that there are no adjustment costs to investment and no depreciation of capital. Legal firms can borrow at interest rate $r > 0$ from world markets. Under the absence of adjustment costs to investment, and given the fact that the number of skilled workers is known one period in advance, the amount of capital in the skilled labor sector is adjusted each period so that $F(K^s_t, L^s_t) = r$. Hence there is a constant capital-labor ratio in this sector, which determines the wage of skilled labor $w^s$ which is constant as well. This wage $w^s$ depends on $r$ and on technology only. Production in the legal unskilled labor sector is described by $Y^n_t = G(L^n_t, N)$ where $Y^n_t, L^n_t$ and $N$ are output, unskilled labor and land respectively. Let the aggregate amount of land be fixed at $N$, so that demand of unskilled labor is

$$G_L(L^n_t, N) = Q(L^n_t)$$

where $Q$ is a function that describes the diminishing marginal productivity of unskilled labor. We assume that all markets are perfectly competitive and expectations are fully rational.

The illegal sector is an abstraction of an organized sector dedicated exclusively to prey on legal workers. It abstracts from the different types of illegal pecuniary
activities that arise in the real world, like robbery in general, burglary, kidnapping, economic extortion etc, but can be understood as having the same end in sight, namely material incentives by preying on legal workers.\textsuperscript{3,4} The organization of the "firms" that operate in this sector is conceptualized in the following manner: members of the organization acquire the income from illegal activities and then share equally with all the other members. This is a strong assumption but simplifies away the hierarchy of the organization that would presumably divides in an unequal fashion the income acquired. The acquisition of the income in the illegal sector by delinquents is described by the following "pseudo production function" which is assumed to be linear in the input labor where delinquents and workers are matched randomly:

\[ E(Y^d_t) = (1 - \pi)\rho(\theta_t W^n_t + \eta_t W^s_t)L^d_t. \] (1)

The term \( E(Y^d_t) \) denotes the expected income that is acquired through delinquency, \( \theta_t \) and \( \eta_t \) are respectively the probabilities of encountering both unskilled and skilled workers in period \( t \), \( L^d_t \) is the labor used in the delinquent sector, \( \rho \in [0, 1) \) represents the fraction of the wealth that a delinquent is able to get from his victims in any given encounter, while \( W^n_t \) and \( W^s_t \) denote the overall wealth levels of unskilled and skilled workers respectively. Since the model has two kind of individuals, namely legal workers and delinquents, then it must be the case that \( \theta_t + \eta_t = 1 - \lambda_t \) where \( \lambda_t \) is the probability in period \( t \) of encountering a delinquent in any given random match. We assume that encounters among delinquents do not generate any net gain for them. With probability \( \pi \in (0, 1) \) the delinquent is apprehended and convicted by law enforcement authorities in which case no wealth is maintained by the delinquent\textsuperscript{5}, while with probability \((1 - \pi)\) a delinquent can obtain a net amount \( \rho(\theta_t W^n_t + \eta_t W^s_t) \) of expected wealth under random matching. We call \( \pi \) the punishment probability.

We can define an average expected "implicit wage" acquired by a delinquent in this economy as \( w^d_t \equiv E(Y^d_t) / L^d_t \) given the assumption of income sharing among members of the illegal sector and therefore one can rearrange (1) to represent \( w^d_t \) as

\[ w^d_t \equiv E(Y^d_t) / L^d_t = (1 - \pi)\rho [(1 - \lambda_t - \eta_t) W^n_t + \eta_t W^s_t]. \] (2)

\textsuperscript{3}We consider delinquency as the illegal activity with the explicit objective of acquiring income or money. In this sense delinquency is a criminal act. Nonetheless, not all types of crimes could be considered delinquent acts. For example, homicides from serial killers do not have a pecuniary motive which in our view would not be considered to be delinquent acts.

\textsuperscript{4}This differs for illegal activities like illegal drugs which are goods that are considered to be illegal but are produced in the same way as legal goods.

\textsuperscript{5}We assume that once a robbery (or kidnap) occurs with probability \( \pi \) law enforcement authorities are able to apprehend and convict the delinquent and give back the wealth seized to the victim at no cost to the victim.
Note that \( w^d_t \) is a decreasing function in \( \lambda_t \) for a given value \( \eta_t \) which means that a higher probability of encountering a delinquent lowers the material incentives for all delinquents in this sector in expected terms. Hence, the illegal sector becomes less attractive when more delinquents enter the sector.

2.2 Preferences and Overlapping Generations

Individuals in this economy live two periods as young and old adults each in overlapping generations. In each generation there is a continuum of individuals of size \( L \). Each individual has just one child (there is no population growth), can work as unskilled in the first period of her life or invest in human capital when young and work as skilled worker when old, or choose a delinquency activity when young. For simplicity we shall assume that all individuals consume when old and only work one period. Unskilled workers and delinquents work when young while skilled workers do so when old. Delinquents enjoy their loot when old if they are not apprehended by law enforcement authorities when young. Moreover, we assume explicitly that decisions are irreversible which implies that a delinquent cannot go back to the legal unskilled sector when old.\(^6\) Individuals that choose to educate themselves invest \( h > 0 \) when young and are able to work in the skilled labor sector when old given that we assume away unemployment.

All individuals consume when old, work one period of their life, care in the same way about their children and lose utility if they choose to lead a delinquent life. This is modelled with a log utility specification in the following way

\[
u = \alpha \log c + (1 - \alpha) \log b - d \log I,\]

where \( 0 < \alpha < 1 \) captures the weight on consumption of an individual, \( c \) is consumption in the second period, \( b \) is the bequest left to his/her child, \( I \) is a psychic cost\(^7\) of committing delinquent acts, \( d = \{0, 1\} \) is a binary variable such that \( d = 1 \) means that an individual chooses to be a delinquent and zero otherwise.

All individuals are born with the same potential abilities, same preferences and psychic cost from engaging in illegal activities. They differ only in the amounts they inherit from their parents in terms of wealth \( x_t \) where \( D_t(x_t) \) is the cumulative

\(^6\)This assumption of irreversibility is strong but Tauchen, Witte and Griesinger (1994) found evidence of a negative relation between studying and/or working with the probability of engaging in criminal activities. They argue that this behavior comes from keeping individuals linked to legal activities through their contact with either an educational or labor institution and not necessarily due to a higher education attainment that brings higher wages.

\(^7\)This psychic cost can represent guilt or shame from committing criminal acts and should correspond to the monetary equivalent.
distribution function of wealth $x_t$ in period $t$ with support $[0, \infty)$. This distribution satisfies $\int_0^\infty dD_t(x_t) = L$.

As argued above we assume the existence of financial markets that allow individuals to save and earn interest on their savings at interest rate $r > 0$ given exogenously by world markets. The financial markets lend these funds to firms that pay interest rate $r$. Nonetheless, we assume an extreme imperfection in the credit market for individual borrowers that want to invest in education, namely that no access to credit is allowed to finance investment in human capital. Hence, individuals born in period $t$ that choose to invest in human capital can do so only if they have enough wealth to pay the investment $h$. This is a working assumption that can be relaxed with less stringent market imperfections in line with Galor-Zeira (1993) without affecting the main results that we find.

### 2.3 Optimal Bequests

Recall $\lambda_t$ denotes the probability in period $t$ for a legal worker to encounter a delinquent. When the encounter occurs the delinquent steals fraction $\rho W_t$ from a worker with overall wealth $W_t$, otherwise the encounter does not occur and the worker loses nothing. Therefore an individual born in period $t$ with wealth $W_t$ chooses $b_t$ in order to maximize expected utility

$$
\max_{b_t} E(U_t) = \alpha [(1 - \lambda_t) \log(W_t - b_t) + \lambda_t \log ((1 - \rho)W_t - b_t)] \\
+ (1 - \alpha) \log b_t - d \log I
$$

We assume that stealing affects directly the consumption of the individual through wealth that is lost since it is equal to $W_t - b_t$ if the individual is not matched with a delinquent and is $(1 - \rho)W_t - b_t$ if matched with one. The first order condition boils down to

$$
\frac{\partial E(U_t)}{\partial b_t} = -\frac{\alpha(1 - \lambda_t)}{(W_t - b_t)} - \frac{\alpha \lambda_t}{(1 - \rho)W_t - b_t} + \frac{1 - \alpha}{b_t} = 0
$$

The resulting equation is a quadratic function in $b_t$ with solution

$$
b_t = W_t \left\{ \frac{\alpha}{2} \left[ B(\lambda_t) - \sqrt{B(\lambda_t)^2 - \frac{4(1 - \alpha)(1 - \rho)}{\alpha^2}} \right] \right\} \equiv W_t \Gamma(\lambda_t)
$$

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8This might be rationalized by assuming that individuals that invest a certain amount in their education through acquiring a credit can leave the economy at zero cost without paying back the loan.
where \( B(\lambda_t) = 1 - \rho(1 - \lambda_t) + \left( \frac{1-\alpha}{\alpha} \right) (2 - \rho) > 0 \) since \( \rho(1 - \lambda_t) < 1 \). Importantly the optimal bequest is a linear function of \( W_t \) and we take the negative root as the solution of the problem\(^9\) showing in the appendix that \( \Gamma'(\lambda) < 0 \) and \( 0 < \Gamma(\lambda) < 1 \) for all \( \lambda \in [0, 1] \) which guarantees that the optimal bequest is always positive. Interestingly the economic interpretation of \( \Gamma'(\lambda) < 0 \) is quite intuitive since it means that the more likely an individual is robbed the less likely she will be able to inherit to her child and therefore the more likely she will consume out of her wealth. This shows how the likelihood of being a delinquent victim affects negatively inheritances.

Replacing \( b_t = W_t \Gamma(\lambda_t) \) in the expected utility function that is maximized in (3) yields the expected life time indirect utility function

\[
EU = \log W_t - d \log I + \varepsilon(\lambda_t)
\]

where \( \varepsilon(\lambda_t) = \alpha(1 - \lambda_t) \log(1 - \Gamma(\lambda_t)) + \lambda_t \log (1 - \rho - \Gamma(\lambda_t)) + (1 - \alpha) \log \Gamma(\lambda_t) \). Note that \( \varepsilon(\lambda_t) \leq 0 \) and furthermore \( \frac{d\varepsilon}{d\lambda} < 0 \). Function (6) proves important to determine the different choices that individuals make.

### 2.4 Occupation Choices and Short-Run Equilibrium

We now turn to describe individual optimal decisions. There are three occupations that individuals can choose: unskilled worker \((n)\), skilled worker \((s)\) and delinquency \((d)\). Overall wealth consists of inherited wealth denoted by \( x \) and income earned during the lifetime of an individual. Therefore the overall wealth levels of unskilled and skilled workers are respectively \( W^n_t \equiv x_t + w^n_t \) and \( W^s_t \equiv x_t + w^s \) for period \( t \). Consider an individual that inherits \( x_t \geq h \) who decides to work as skilled \((d = 0)\) and invest in human capital, her lifetime indirect expected utility and bequest are respectively

\[
EU^s(x_t) = \log [(w^s + (x_t - h)(1 + r))] + \varepsilon(\lambda_t)
\]

\[
b^s(x_t) = [(w^s + (x - h)(1 + r)]\Gamma(\lambda_t).
\]

Consider now an individual who inherits an amount \( 0 < x_t < h \) of wealth in her first period of life and decides to work as unskilled \((d = 0)\) and not invest in human capital then her lifetime indirect expected utility and bequest are

\[
EU^n(x_t) = \log [(x_t + w^n_t)(1 + r)] + \varepsilon(\lambda_t)
\]

\[
b^n(x_t) = [(x_t + w^n_t)(1 + r)]\Gamma(\lambda_t).
\]

\(^9\)This is due to the economic intuition of the solution which shall be explained below.
Alternatively, an individual who inherits an amount \(0 \leq x_t < h\) of wealth in his first period of life and decides to become a delinquent \((d = 1)\) loses utility \(\log I\) for certain and has lifetime expected utility and bequest

\[
EU^d(x_t) = \log \left( (x_t + w^n_t) (1 + r) \right) - \log I + \varepsilon(\lambda_t)
\]

\[
b^d(x_t) = \left( (x_t + w^n_t) (1 + r) \right) \Gamma(\lambda_t).
\]

Since occupational choices are irreversible once taken then a delinquent which chooses this occupation when young cannot become a skilled worker since he forgoes the opportunity of investing in education in her first period of life. Hence, no educated delinquents can arise in the model. Moreover, if the wage differential between skilled and unskilled is sufficiently wide, taking into account the investment cost \(h\), all legal workers would prefer to work as skilled. To see this notice that \(EU^s(x_t) \geq EU^n(x_t)\) is true if and only if

\[
w^s - h(1 + r) \geq w^n_t(1 + r)
\]

for every \(t\). We assume that (7) holds true for every value of \(w^n_t\) otherwise there would be no incentive to invest in human capital. Nonetheless, the possibility of gaining access to education depends on inherited wealth since individuals with inherited wealth \(x_t\) strictly less than \(h\) cannot educate themselves given that it has been assumed away any possibility for financing this investment with future earnings.

Individuals with inherited wealth less than \(h\) have to decide between working as an unskilled worker or becoming a delinquent. Individuals prefer to work as legal unskilled workers relative to becoming a delinquent as long as \(EU^n(x_t) \geq EU^d(x_t)\), that is as long as

\[
(x_t + w^n_t)I \geq x_t + w^d_t.
\]

Note from (2) that \(w^d_t = (1 - \pi) \rho \left[ (1 - \lambda_t - \eta_t) W^n_t + \eta_t W^s_t \right]\) and by construction \(W^n_t \equiv x_t + w^n_t\) and \(W^s_t \equiv x_t + w^s_t\). Replacing these in (8) yields a threshold wealth level as a function of \(\lambda_t\) and \(w^n_t\) expressed as

\[
x_t \geq f(\lambda_t, w^n_t) = \max \left\{ 0, \frac{(1 - \pi) \rho \left[ (1 - \lambda_t - \eta_t) w^n_t + \eta_t w^s_t \right] - w^d_t I}{I - 1 - (1 - \pi) \rho (1 - \lambda_t)} \right\}
\]

\[10\]We have assumed in this calculation that an individual considers himself as negligible when becoming a delinquent in the sense that he does not think he will vary the fraction of delinquents in the economy. This implies that the term \(\varepsilon(\lambda_t)\) can safely be eliminated on both sides of the inequality.
We assume that $I \geq 2$ from now on which implies that the denominator in (9) is positive while if $I$ is large enough under a small wage gap the numerator can be negative which explains the max operator.

We turn now on the determination of the unskilled equilibrium wage $w^n_t$ at time $t$. The supply of unskilled labor depends on the wealth distribution of the economy since those who have a wealth level between $f(\lambda_t, w^n_t)$ and $h$ choose to be unskilled workers given that we shall assume throughout that $f(\lambda_t, w^n_t) \leq h$. Hence, since each individual has one unit of labor each period the supply function of unskilled labor in period $t$ is given by

$$S_{n,t} = L^n_t = \int_{f(\lambda_t, w^n_t)}^{h} dD_t(x_t)$$

(10)

Competitive markets in the unskilled sector equate aggregate demand and supply of unskilled labor i.e. $S_{n,t} = Q(\bar{N})$ to determine the unskilled wage $w^n_t$ in each period. Given that the aggregate demand is fixed in any given period for a given value of $\bar{N}$ this unskilled equilibrium wage depends negatively on the fraction of unskilled workers in the economy i.e. $w^n_t = w^n(\theta_t, \bar{N})$ such that $\frac{\partial w^n_t}{\partial \theta_t} < 0$. Importantly it increases with the level of land in the economy i.e. $\frac{\partial w^n_t}{\partial \bar{N}} > 0$ given that this would shift demand to the right and for the same supply of workers the unskilled wage must increase. For future reference define $\bar{w}^n \equiv w^n(0, \bar{N})$ as the highest feasible unskilled wage when no individual would supply unskilled labor and $\underline{w}^n \equiv w^n(1 - \eta_t, \bar{N})$ as the lowest yet positive unskilled wage when all labor of individuals with less than $h$ is allocated to the legal unskilled sector.

Figure 1 illustrates both the demand and supply of unskilled labor such that at their intersection the unskilled wage is determined. Notice that at $w^n_t = w_s/(1 + r) - h$ individuals are indifferent between investing in human capital and working as unskilled, hence the supply curve is upward sloping but becomes flat at this wage. Nonetheless, it can contain vertical segments.
The amount an individual inherits in her first period of life, therefore, fully determines her decisions whether to invest in human capital or work as unskilled or become a delinquent, and how much to consume and bequeath. Hence, the distribution $D_t$ determines economic performance in period $t$: the amount of skilled labor $L^s_t = \int_h^\infty dD_t(x)$, delinquency $L^d_t = \int_0^{f(\lambda_t,w^p_t)} dD_t(x)$ and unskilled labor $L^n_t = \int_{f(\lambda_t,w^p_t)}^{h} dD_t(x)$.

Rational expectations require consistency of expectations and chosen occupations such that the following are satisfied

\begin{equation}
\eta_t = \frac{\int_h^\infty dD_t(x_t)}{L}; \quad \theta_t = \frac{\int_0^{f(\lambda_t,w^p_t)} dD_t(x_t)}{L}; \\
\lambda_t = (1 - \psi \pi) \frac{\int_0^{f(\lambda_t,w^p_t)} dD_t(x_t)}{L}
\end{equation}

where the fraction $\psi \pi \frac{\int_0^{f(\lambda_t,w^p_t)} dD_t(x_t)}{L}$ represents the fraction of delinquents that are apprehended and effectively convicted in period $t$ under random matching. Here we assume that not all apprehended delinquents can be convicted since having enough "evidence" to incriminate the suspected felon is something that is not always feasible. We assume that only a fraction $\psi \in (0,1)$ of apprehended delinquents are effectively found guilty of the crime they are accused of beyond a reasonable doubt. This rationalizes that law enforcement authorities can incapacitate effectively at most $\psi$ of the fraction of apprehended delinquents in a given period by actually putting them
in jail.\footnote{Importantly individuals that are put in jail in period $t$ do not circulate in the economy in that period therefore they are modelled here "as if" they disappeared or vanished in the distribution of wealth for (only) period $t$. They could still have children in jail so the population growth is zero at all times.} This motivates the following definition.

**Definition 1** A short run rational expectations equilibrium (SREE) of the economy described above consists of a distribution of fractions $\nu_t = [\lambda_t, \theta_t, \eta_t]$ for period $t$ where $\lambda_t + \theta_t + \eta_t = 1$ such that in period $t$ individuals choose occupations that maximize expected utility, firms have zero profits, markets balance and conditions (11) are met.

The following theorem gives the conditions to secure the existence and uniqueness of the fraction of delinquency in the economy.

**Theorem 1** If the economy described above satisfies (7) for all $w^n_t$ and the distribution of wealth $D_t$ generates an equilibrium unskilled wage such that $w^n_t \in [w^n_t, M_1] \neq \emptyset$ then it has a unique SREE with $\lambda_t \in (0, 1 - \eta_t]$ for any given $t$. Otherwise $\lambda_t = 0$.

**Proof.** Firms have zero profits in equilibrium given the assumption of constant returns to scale in both legal sectors. Individuals maximize expected utility and choose optimally bequests and occupations in period $t$ given the threshold values $h$ and $f(\lambda_t, w^n_t)$. To establish the existence of a SREE one has to establish the existence of $\lambda_t \in [0, 1 - \eta_t]$ that satisfies (11) recognizing that the cutoff wealth level $f(\lambda_t, w^n_t)$ is a function of $\lambda_t$ for given $\eta_t$ from (9). Since by definition $\theta_t = 1 - \lambda_t - \eta_t$ and $\eta_t = \frac{\int_0^\infty D_t(x_t) \, dx_t}{L} > 0$ is given for any $t$ we have that an increase in $\lambda_t$ is a proportional decrease in $\theta_t$. Since the equilibrium unskilled wage satisfies $\frac{\partial w^n_t}{\partial \theta_t} < 0$ then it must be the case that $\frac{\partial w^n_t}{\partial \lambda_t} > 0$. Consequently, define the following continuous function in $\lambda_t$

$$g(\lambda_t) \equiv \lambda_t - (1 - \psi\pi) \frac{\int f(\lambda_t, w^n_t(\lambda_t)) \, dD_t(x_t)}{L}$$

in the support $[0, 1 - \eta_t]$. Note that evaluating at zero yields

$$g(0) = - (1 - \psi\pi) \frac{\int f(0, w^n_t) \, dD_t(x_t)}{L}$$

which is zero if $f(0, w^n_t) \leq 0$ or equivalently if $w^n_t \geq \frac{(1-\pi)\rho_0 w^r}{f(I(1-\pi)\rho(1-\eta_t))} \equiv M_1$ where $M_1 > 0$ given that $I \geq 2$. Now $g(0) < 0$ if $f(0, w^n_t) > 0$ arises or equivalently $w^n_t < M_1$ is satisfied. Furthermore under (7) we have $\bar{w}^n_t = \frac{w^r}{(1+r)} - h$ such that

$$g(1 - \eta_t) = 1 - \eta_t - (1 - \psi\pi) \frac{\int f(1 - \eta_t, \bar{w}^{n_t}) \, dD_t(x_t)}{L} > 0$$
which holds since the fraction of skilled workers and delinquents that are not captured by law enforcement authorities cannot exceed one. The continuity of \( g(\cdot) \) establishes that there exists a \( \lambda_t \) that satisfies

\[
\lambda_t = (1 - \psi t) \frac{\int_0^{f(\lambda_t, w^n_t(\lambda_t))} dD_t(x)}{L}.
\]

Moreover note that by Leibniz rule\(^\text{12}\)

\[
g(\lambda_t) = 1 - (1 - \psi t) \left[ f_1 + f_2 \frac{\partial w^n_t}{\partial \lambda_t} \frac{d_t(f(\lambda_t, w^n_t(\lambda_t)))}{L} \right] > 0
\]

since \( f_1 \leq 0, \ f_2 = (1 - \pi) \rho \theta_t - I \leq 0 \) and \( \frac{\partial w^n_t}{\partial \lambda_t} > 0 \) where \( d_t(f(\lambda_t, w^n_t(\lambda_t))) \) is the density function of \( D_t \) evaluated at \( f(\lambda_t, w^n_t(\lambda_t)) \) which is always positive. Hence, the SREE is unique for each \( t \). \( \blacksquare \)

Note that a positive fraction of delinquency in equilibrium arises in the SREE if the distribution of wealth \( D_t \) generates an equilibrium unskilled wage such that \( w^n_t \in [w^n, M_1) \neq \emptyset \) i.e. if there is a high enough wealth inequality such that the wealth distribution entails a sufficiently low unskilled wage in equilibrium. This amounts to say that if there is sufficient amount of poor unskilled workers delinquency arises in equilibrium. On the other hand, if the wealth distribution is such that the unskilled wage is above \( M_1 \) then delinquency does not arise in equilibrium. This implies that the less wealthy households in period \( t \) are the ones self-selected into delinquency when high wealth inequality arises and the unskilled wage is low enough which entails a link between poverty, wealth inequality and delinquency in the short run.

2.5 The Dynamics of Wealth Accumulation and the Poverty Trap

The distribution of wealth not only determines equilibrium in period \( t \), but also determines next period distribution of inheritances through the following dynamic equation:

\[
x_{t+1} = \begin{cases} 
  b^d(x_t; \lambda_t) = \left( (x_t + w^n_t) (1 + r) \right) \Gamma(\lambda_t) & \text{if } 0 \leq x_t < f_t \\
  b^s(x_t; \lambda_t) = \left( (x_t + w^n_t)(1 + r) \right) \Gamma(\lambda_t) & \text{if } f_t \leq x_t < h \\
  b^s(x_t; \lambda_t) = \left( ((x_t - h)(1 + r) + w^s) \Gamma(\lambda_t) \right) & \text{if } x_t \geq h 
\end{cases}
\]

where for simplicity we denote \( f_t \equiv f(\lambda_t, w^n_t(\lambda_t)) \). As seen above individuals who have \( x \) greater or equal than \( h \) choose to become skilled workers, those with \( x \) less

\(^{12}\text{Recall } \frac{\partial}{\partial z} \int_{a(z)}^{b(z)} f(x, z) \, dx = \int_{a(z)}^{b(z)} \frac{\partial f}{\partial z} \, dx + f(b(z), z) \frac{\partial b}{\partial z} - f(a(z), z) \frac{\partial a}{\partial z}.\)
than $f_t$ choose delinquency while individuals who inherit between $f_t$ and $h$ choose to become unskilled workers. Using $\tilde{\pi}^n = b^n(\tilde{\pi}^n; \lambda_t)$ in (12), $\Gamma'(\lambda_t) < 0$ and assuming from here onwards a sufficient condition $(1 + r)\Gamma(0) < 1$ one has wealth level $\tilde{x}_n$ well defined given by

$$\tilde{x}_n(\lambda_t) = \frac{(1 + r)w^n_t}{\Gamma(\lambda_t) - (1 + r)}$$

which is positive and where $\lambda_t \in [0, 1 - \eta_t]$ is a SREE fraction of non apprehended delinquents. Using $\tilde{x}^s = b^s(\tilde{x}^s; \lambda_t)$ in (12) and again under $\Gamma'(\lambda_t) < 0$, $(1 + r)\Gamma(0) < 1$ one has wealth level $\tilde{x}^s$ given by

$$\tilde{x}^s(\lambda_t) = \frac{w^s_t - h(1 + r)}{\Gamma(\lambda_t) - (1 + r)}.$$  

Under assumption (7) we have that $\tilde{x}^s(\lambda_t) \geq \tilde{x}^n(\lambda_t)$ for all $\lambda_t \in [0, 1 - \eta_t]$.

Note that wealth level $\tilde{x}^s(\lambda_t)$ is decreasing in $\lambda_t$ given that $\Gamma'(\lambda) < 0$ while $\tilde{x}^n(\lambda_t)$ can be increasing or decreasing since $\frac{\partial w^n_t}{\partial \lambda_t} > 0$ while $\Gamma'(\lambda) < 0$. It turns out that if the elasticity of unskilled wages with respect to $\lambda$ is large enough such that

$$\varepsilon_{w^n, \lambda} \equiv \frac{\frac{\partial w^n_t}{\partial \lambda_t} \lambda_t}{w^n_t} > \frac{-\lambda_t \Gamma'}{[1 - (1 + r)\Gamma] \Gamma},$$

then we have that $\frac{\partial \tilde{x}^n}{\partial \lambda_t} > 0$. The right hand side threshold $\frac{-\lambda_t \Gamma'}{[1 - (1 + r)\Gamma] \Gamma}$ is non negative under $\Gamma'(\lambda) < 0$, $\lambda_t \in [0, 1 - \eta_t]$ and $(1 + r)\Gamma(0) < 1$. Assumption (15) simply states that $\Gamma'$ is not so sensitive to changes in $\lambda$ relative to the sensitivity in $\lambda$ of the equilibrium unskilled wage. In other words, as $\lambda$ increases $\theta$ decreases making the unskilled wage rise off setting more than enough the fall in $\Gamma$.

Figure 2 illustrates a typical configuration of the short run dynamics of wealth accumulation in the economy given by (12). The points in which the curve intersects with the 45 degree line correspond to $\tilde{x}^n$ and $\tilde{x}^s$ for a SREE value $\lambda_t$. Individuals with wealth levels less than $h$ (including unskilled and delinquents) would move in the short run towards $\tilde{x}^n$ while those with wealth level greater than $h$ move towards $\tilde{x}^s$. Nonetheless, these wealth levels depend explicitly on $\lambda_t$ and the dynamics of wealth accumulation should not be considered the long run steady state wealth levels since one would have to determine within the dynamic system the value $\lambda_\infty \equiv \lim_{t \to \infty} \lambda_t$ to which $\lambda_t$ converges in the long run.
Let us examine the long run behavior of the dynamic equation (12). From (9) one can see that the cutoff point $f_t$ and loot $w^d_t$ decrease with $\lambda_t$ while $\pi^n$ increases under assumption (15). Hence in Figure 2 where $f_t < \pi^n$ is satisfied in period $t$, as the dynamics step in $f_{t+1}$ is higher as a non-negligible fraction of non apprehended delinquents migrate from the illegal sector towards the legal unskilled sector decreasing $\lambda_{t+1}$. The wealth level $\pi^s$ increases necessarily as $\lambda_t$ decreases while $\pi^n$ decreases under assumption (15). Hence, as the economy in Figure 2 moves in time delinquency decreases while the wealth gap $\pi^s - \pi^n$ increases. This motivates two cases to consider: i) a vanishing fraction of delinquents such that $\lambda_\infty = 0$ and ii) persistent delinquency $\lambda_\infty > 0$. If $\lambda_\infty = 0$ then one has a long run behavior as in the Galor-Zeira model abstracting from credit markets for households. Nonetheless, we argue below that in the long run it is possible to have $\lambda_\infty > 0$ under certain conditions. Regardless of either case this convergence process requires us to consider a steady state in which $\lambda_\infty \equiv \lim_{t \to \infty} \lambda_t$. Consequently a steady state in the dynamics of wealth accumulation such that $\lambda_\infty > 0$ requires the migration outflow to be exactly the migration inflow to the delinquent sector. This motivates the following definition.

**Definition 2** A long run rational expectations equilibrium (LREE) consists of a SREE in which $\lim_{t \to \infty} \pi^i (\lambda_t) = \pi^i (\lambda_\infty)$ for $i = n, s$, and the long run wealth threshold $f_\infty$ satisfies $f_\infty = \pi^n (\lambda_\infty)$ if $\lambda_\infty \in (0, 1 - \eta_t]$ or $f_\infty < \pi^n (0)$ if $\lambda_\infty = 0$. 

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To get an intuition for this definition consider Figure 2 and let us focus on the dynamics of the bequest functions $b^d(x_t; \lambda_t)$ and $b^n(x_t; \lambda_t)$ as time evolves. Since the process starts off such that $f_t < \overline{\pi}_n(\lambda_t)$ then some fraction of the offspring of (non apprehended) delinquent households cross $f$ (namely those with wealth level arbitrarily close to $f_t$) and enter the legal unskilled sector inducing a decrease in $\lambda_{t+1}$.\(^{13}\) This in turn increases the threshold $f_{t+1}$, loot $w^d_{t+1}$ while decreasing $\overline{\pi}_n$ under assumption (15). The net effect is that $\lambda$ should eventually decrease weakly so long as $f_s < \overline{\pi}_n(\lambda_s)$ for some $s > t$. This process continues up to the point in which equality $f_\infty = \overline{\pi}_n(\lambda_\infty)$ occurs consistent with persistent delinquency $\lambda_\infty \in (0, 1 - \eta_t]$. Nonetheless it could happen that delinquency vanishes before this equality is reached i.e. $f_\infty < \overline{\pi}_n(0)$ consistent with $\lambda_\infty = 0$. On the other hand a similar logic occurs for the case in which the dynamic process starts off with $f_t > \overline{\pi}_n(\lambda_t)$. In this case delinquency increases as more households are induced by the dynamics around $\overline{\pi}_n$ to enter the delinquent sector increasing $\overline{\pi}_n(\lambda_t)$ under assumption (15) and eventually decreasing $f_t$. If $\bar{w}^n = w_s/(1 + r) - h$ then no delinquency arises since in that case a delinquent would earn the same as a skilled worker but would have to suffer a psychic cost of $I$. Hence, not all unskilled labor could choose a delinquent life and therefore $\lambda_\infty$ has to be lower than $1 - \eta_t$.

Figure 3 illustrates the limiting behavior of the dynamic system where the thin line

\[^{13}\]This is because dynastic delinquent households as they accumulate wealth would cross eventually the threshold $f_s$ for some period $s > t$ given that they are only delayed some finite number of periods by some law enforcement detentions.
is consistent with the case \( f_\infty = \bar{\pi}_n (\lambda_\infty) \) for persistent delinquency \( \lambda_\infty \in (0, 1 - \eta_t) \). Note that \( h \geq \bar{\pi}_n (\lambda_\infty) \) arises in the long run since we have assumed \( h \geq f_t \) for all \( t \) and this entails a poverty trap which induces persistent inequality and eventually delinquency \( \lambda_\infty > 0 \) in the long run. To see this note that apprehended and convicted delinquent households that have just \( f_\infty = \bar{\pi}_n (\lambda_\infty) \) (or \( \varepsilon \) less of wealth) in the long run will not be able to increase their wealth in \( w_\infty^d \) forcing them to leave a bequest less than \( \bar{\pi}_n (\lambda_\infty) \) for their offspring, given that they still have to consume when adults. Hence, these offspring would necessarily choose again and again to become delinquents consistent with having persistent delinquency in the economy. On the other hand, non apprehended delinquents that have just \( f_\infty = \bar{\pi}_n (\lambda_\infty) \) (or \( \varepsilon \) less of wealth) would be able to secure loot \( w_\infty^d \) allowing them to bequest a wealth level greater than \( \bar{\pi}_n (\lambda_\infty) \). Hence, their offspring would choose to become legal workers in the next period. Nonetheless, these households due to the existence of a poverty trap would eventually end up having again \( f_\infty = \bar{\pi}_n (\lambda_\infty) \) in the long run and therefore could end up having some offspring that would choose to become delinquents. It is this outflow and inflow of individuals from and to the illegal sector that would have to be balanced off in the long run consistent with a LREE such that \( \lambda_\infty > 0 \) remains constant. Figure 4 illustrates a bimodal pdf wealth distribution \( d_\infty \) consistent with this dynamic process and corresponds to the thin line of Figure 3.

Moreover, to get persistent delinquency in the long run one requires additionally that \( b^d(f_\infty; \lambda_\infty) < h \). To see why consider what would happen if we had \( b^d(f_\infty; \lambda_\infty) \geq \)
$h$. In this case the offspring of non-apprehended delinquent households with wealth level $f_\infty$ would inherit enough wealth to educate themselves leapfrogging over the poverty trap and entering eventually the skilled sector. Hence, in the long run $\lambda_\infty = 0$. To get persistent delinquency one then requires $b^d(f_\infty; \lambda_\infty) < h$. It remains to show that under certain conditions there exists a LREE with persistent delinquency.

**Theorem 2** If the economy described above satisfies $h \geq I\bar{\pi}^n(1 - \eta_t)$ as well as assumptions (15), (7) for all $w^n_1$ such that the initial wealth distribution $D_0$ generates $w^n_0 \in [w^n_1, M_2) \neq \emptyset$ then there exists a unique LREE of the economy described above such that $\lambda_\infty \in (0, 1 - \eta_t)$.

**Proof.** Consider a SREE and note that assumptions $h \geq I\bar{\pi}^n(1 - \eta_t)$, $w^n > 0$ and $(1 + r)\Gamma(0) < 1$ implies that $\pi_n > 0$ intersects the 45 degree line and is bounded away from infinity generating a poverty trap since otherwise the $b^n$ function would not intersect the 45 degree line. Assumption (7) guarantees that $\bar{\pi}^n(\lambda) \geq \bar{\pi}^n(\lambda)$. Define the following function on the domain $[0, 1 - \eta_t]$ 

$$m(\lambda) = f(\lambda, w^n(\lambda)) - \bar{\pi}^n(\lambda)$$

which is a continuous function of $\lambda$ given that both $w^n$ and $\Gamma(\lambda)$ are continuous in $\lambda$. Note that $m'(\lambda) < 0$ since $f'(\lambda) < 0$ and $\frac{\partial \bar{\pi}^n}{\partial \lambda} > 0$ under assumption (15). Moreover, $f(0, w^n) > \bar{\pi}^n(0)$ arises when $w^n < M_2$ where $M_2 \equiv \frac{(1 - \pi)\rho(1 - \eta_t)}{I - (1 - \pi)(1 - \eta_t) + \frac{1 - (1 - \pi)\rho}{1 + 1 - \eta_t}} < M_1$. Hence $m(0) > 0$ and $m'(\lambda) < 0$ which generates persistent delinquency since $m(\lambda_\infty) = 0$ must involve $\lambda_\infty \in (0, 1 - \eta_t)$. On the other hand if, $w^n \geq M_2$ one would have $m(0) \leq 0$ and the long run steady state is compatible with $\lambda_\infty = 0$. We still need to check that $b^d(f_\infty; \lambda_\infty) < h$ holds. Assumption $h \geq I\bar{\pi}^n(1 - \eta_t)$ implies that $h > I\bar{\pi}^n(\lambda_\infty)$ which can be rewritten as

$$w^n(\lambda_\infty) \left[ \frac{1}{1 - (1 + r)\Gamma(\lambda_\infty)} \right] < \frac{h}{I(1 + r)\Gamma(\lambda_\infty)}$$

$$w^n(\lambda_\infty) \left[ \frac{(1 + r)\Gamma(\lambda_\infty)}{\Gamma(\lambda_\infty) - (1 + r)} \right] < \frac{h}{I(1 + r)\Gamma(\lambda_\infty) - w^n(\lambda_\infty)}$$

$$f_\infty = \pi_n(\lambda_\infty) < \frac{h}{I(1 + r)\Gamma(\lambda_\infty) - w^n(\lambda_\infty)}$$

since in LREE with $\lambda_\infty \in (0, 1 - \eta_t)$ we have $f_\infty = \pi_n(\lambda_\infty)$. Moreover

$$(f_\infty + f_\infty(I - 1) + w^n(\lambda_\infty) I) < \frac{h}{(1 + r)\Gamma(\lambda_\infty)}$$

$$f_\infty + w^n_d < \frac{h}{(1 + r)\Gamma(\lambda_\infty)}$$
since from (2) \(w^d_\infty = f_\infty (I - 1) + w^n (\lambda_\infty) I\). Note that this last expression rearranged corresponds to \(b^d(f_\infty; \lambda_\infty) = (f_\infty + w^d_\infty) (1 + r) \Gamma(\lambda_\infty) < h\). Hence, \(b^d(f_\infty; \lambda_\infty) < h\) is satisfied. ■

Some remarks are in order.

i) Assumption \(h \geq I\pi^n (1 - \eta_t)\) allows for the existence of the poverty trap which in turn makes it more likely that persistent delinquency arises in the long run. This is because \(h\) sufficiently large relative to \(I\pi^n\) implies that the \(b^n\) function intersects the 45 degree line. Moreover, this assumption is crucial to have \(b^d(f_\infty; \lambda_\infty) < h\).

ii) Condition \(w^n_0 \in [w^n, M_2) \neq \emptyset\) implies that persistent delinquency requires that wealth distribution initially generates an unskilled equilibrium wage low enough for delinquent incentives to arise. Moreover, the threshold value \(M_2\) satisfies \(M_2 < M_1\) which shows that for persistent delinquency to arise one needs sufficient mass of poor unskilled workers in the sense of the equilibrium unskilled wage to be low enough (lower than \(M_2\)). This implies that poverty coupled with wealth inequality is a necessary condition for delinquency in the short run but no sufficient for persistent delinquency in the long run unless there is a poverty trap.

Consider Figure 3 again and let us focus on the thin line that represents the steady state wealth distribution compatible with \(\lambda_\infty \in (0, 1 - \eta_t)\) such that \(f_\infty = \pi_n (\lambda_\infty)\). The outflow migration from the illegal delinquent sector to the legal unskilled one should be just the same as the inflow migration from the former to the latter. Hence, we have a continuous flow of households leaving for some periods the illegal sector just to come back eventually to it due to the poverty trap. So it is perfectly possible to have dynastic households that go in and out of delinquency infinitely many times. This circular flow is maintained because of the condition \(b^d(f_\infty; \lambda_\infty) < h\) that does not allow delinquent households to leapfrog over the poverty trap. This intuition is illustrated also in Figure 4 where in the long run the wealth distribution around the poverty trap is not a single point but a region.

3 Comparative Dynamics

In this section we study the dynamic behavior of the economy when some parameters shift. Let us consider technological shocks to productivity in the skilled sector \((w^s)\), restrictions to the land asset in the unskilled sector \((N)\), changes in the punishment probability \((\pi)\), decrease in the human capital investment \((h)\) and finally a continuous technological innovation in the unskilled legal sector.

Let us suppose the economy is in its long run REE such that \(f_\infty = \pi_n (\lambda_\infty)\) consistent with \(\lambda_\infty \in (0, 1 - \eta_t)\). Consider first increasing \(w^s\) due to a possible temporal
one time exogenous technological shock. Let us trace out the effect within the model. In this case initially $f$ is increased in the short run above $\bar{x}_n$ making the illegal sector attractive for individuals since $w^d$ is shifted upward which increases subsequently $\lambda$ in the next period. Hence, as $\lambda$ increases the $\bar{x}_n$ wealth level also increases under assumption (15) while this influx of delinquents decreases the initial rise in $f$ and therefore the dynamics yield in the long run $f_\infty = \bar{x}_n$ again to the same wealth level before the shock. Hence, a temporal increase in $w^s$ produces an outburst of delinquency that eventually dies out later. If the shock is permanent the logic is the same but there is a permanent increase in $\lambda_\infty$ since there is a permanent increase in the incentives to enter the illegal sector. In this case the long run wealth level $\bar{x}_s$ does not go back to the initial one while $f_\infty = \bar{x}_n$ increases permanently.

Consider now a permanent decrease in the level of aggregate land $\bar{N}$. In this case there is an inward shift inward of the labor demand in the unskilled labor sector reducing the equilibrium unskilled wage $w^u$. From (13) one sees that $\bar{x}_n (\lambda_\infty)$ is decreased in the short run under assumption (15). Since $f > \bar{x}_n$ arises there is an incentive to enter the delinquent sector generating an increase in $\lambda$ for the next period. From (14) one sees that this in turn decreases $\bar{x}_s$ due to $\Gamma' < 0$. Now the influx of delinquents makes $f$ decrease while increasing $\bar{x}_n$ from the level after the shock to reestablish $f_\infty = \bar{x}_n$ in the long run. Since $f$ has decreased to a lower wealth level we conclude that the over flow of delinquency reduces both $\bar{x}_s$ and $\bar{x}_n$ permanently impoverished the economy.

Let us consider now a permanent increase in the punishment probability $\pi$. In this case there is a decrease initially in the threshold value $f$ since the expected loot decreases and from (9) one sees that $f$ is a decreasing function in $\pi$. Since $f < \bar{x}_n$ arises there is an incentive for delinquents to leave the illegal sector towards the unskilled labor sector generating a subsequent decrease in $\lambda$ for the next period which in turn decreases the equilibrium unskilled wage $w^u$ since $\frac{\partial w^u}{\partial \lambda} > 0$. Hence, $\bar{x}_n$ decreases in the short run under assumption (15). Moreover, $\bar{x}_s$ is increased due to the lower level of $\lambda$ given that $\Gamma' < 0$. The initial fall in $f$ is attenuated by the subsequent decrease in $\lambda$ while the dynamics reestablish $f_\infty = \bar{x}_n$ in the long run at a lower level than before the change in $\pi$. Hence, a permanent increase in $\pi$ decreases permanently the delinquency of the economy under assumption (15). Nonetheless, this generates a permanent increase in wealth inequality since $\bar{x}_s - \bar{x}_n$ increases. Hence, a permanent

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14 Instead of thinking this as land destruction a more reasonable scenario would be forced displacement of poor households from rural areas towards urban areas because of violence which has occurred for example all too recently in Colombia.

15 This increase can be thought as a higher efficiency level of apprehension or conviction of delinquents due to a technological innovation.
increase in \( \pi \) is associated with an increase in inequality due to the existence of the poverty trap.

Let us consider now a \textit{permanent} decrease in the investment cost \( h \). This could be rationalized as the appearance of public schooling (adjusting for quality) that makes human capital investment cheaper for the population in a world in which private schooling was previously the only option. In this case the decrease in \( h \) during the current period would imply an increase in fraction of skilled labor \( \eta \) for the next period and a corresponding decrease in the fraction of unskilled labor \( \theta \). This in turn generates an increase in the equilibrium unskilled wage \( w^n \) in the short run while also increasing \( \pi^n \). On the other hand, the change in \((\eta, \theta)\) generates a change in \( f \). From (9) deriving with respect to \( \eta \) such that \( \eta \) is increased while \( \theta \) is decreased in the same proportion and \( \theta + \eta = 1 - \lambda \) is unchanged generates

\[
\frac{\partial f}{\partial \eta} = \frac{[w^s - w^n] (1 - \pi) \rho - \frac{\partial w^n}{\partial \theta} [I - (1 - \pi) \rho \theta]}{I - 1 - (1 - \pi) \rho (1 - \lambda)} > 0
\]

since \( \frac{\partial w^n}{\partial \theta} < 0 \), \( w^s > w^n \) and \( I \geq 2 \). Hence, both \( f \) and \( \pi^n \) increase with \( \eta \). This implies an ambiguous effect con \( \lambda \) in the long run since depending on whether \( f \) is greater or not with respect to \( \pi^n \) after the increase in \( \eta \) the delinquency fraction could either increase or decrease. If \( f > \pi^n \) then delinquency increases in the long run while if \( f < \pi^n \) then delinquency diminishes.

Finally, let us consider a continuous improvement in the unskilled labor technology \( G \). In this case the poverty trap ceases to exist eventually and the economy converges to \( \bar{x}^s (0) \). This implies that in the medium run there would be club convergence as in Figure 4 but in the long run there would be absolute convergence towards \( \bar{x}^s (0) \). This process would be similar to the one described in Galor (1996).

\section{Conclusions}

Delinquency seems more persistent than one might think in both developed as well as under developed economies. We study an overlapping generations model under perfect competition similar to Galor-Zeira (1993) which allows us to explore the theoretical linkages between poverty traps, economic inequality and educational attainment. It takes seriously the idea that delinquents choose rationally a criminal life when there is a lack of opportunities to enter a skilled sector that requires previously to attain a certain level of education. It builds on a dual economy in which delinquents are seen as parasites that prey on legal workers. We characterize the optimal bequest of dynastic households in three occupational activities (delinquency, unskilled and skilled workers) that emerge which govern and are determined by the wealth accumulation of
the economy. We show conditions under which a short run delinquency fraction exists and define a steady state of the dynamic system compatible with the possibility of persistent delinquency in the long run. We find that for given levels of law enforcement measures of deterrence and incapacitation delinquency is persistent in the long run if the unskilled equilibrium wage is low enough due to an initially high mass of unskilled workers that coupled with high inequality induces delinquent opportunities.

We study comparative dynamics of the model and show that temporal technological shocks that increase skilled wages or reduces land for the unskilled increase delinquency in the short run producing an outburst of delinquency that dies out later on. If the shock is permanent then the delinquency increases permanently in the long run. We find that permanent increases in law enforcement deterrence and incapacitation policies increase wealth inequality in the long run due to the presence of the poverty trap. We find furthermore that a permanent increase in the fraction of skilled workers (due to a subsidy for human capital investment) has an ambiguous effect on long run delinquency since the incentives for delinquency increase but at the same time the equilibrium unskilled wage increases.

Further research would be to allow for unemployment in the skilled sector and to trace out the effect on delinquent incentives. Another extension could be to generalize the model to consider illegal activities such as narcotics or gambling that are not necessarily seen as preying on workers but more as activities that sell workers services that are illegal in the economy.
Appendix

Proposition 1 Under the assumptions of the model $\Gamma'(\lambda) < 0$ and $0 < \Gamma(\lambda) < 1$ for all $\lambda \in [0, 1]$.

Proof. First we show that $\Gamma'(\lambda) < 0$ for all $\lambda \in [0, 1]$. From (5) differentiating with respect to $\lambda$ we get

$$
\Gamma'(\lambda) = \frac{\alpha}{2} \left[ \rho - \frac{1}{2} \left( B(\lambda)^2 - \frac{4(1-\alpha)(1-\rho)}{\alpha^2} \right)^{-\frac{1}{2}} (2B(\lambda)\rho) \right]
$$

since $B'(\lambda) = \rho$. It is sufficient to show that

$$
1 < \left( B(\lambda)^2 - \frac{4(1-\alpha)(1-\rho)}{\alpha^2} \right)^{-\frac{1}{2}} B(\lambda).
$$

which is satisfied since

$$
\left( B(\lambda)^2 - \frac{4(1-\alpha)(1-\rho)}{\alpha^2} \right)^{\frac{1}{2}} < B(\lambda)
$$

$$
B(\lambda)^2 - \frac{4(1-\alpha)(1-\rho)}{\alpha^2} < B(\lambda)^2
$$

$$
\frac{4(1-\alpha)(1-\rho)}{\alpha^2} > 0.
$$

We have used the fact that $B(\lambda)^2 - \frac{4(1-\alpha)(1-\rho)}{\alpha^2} > 0$ for all $\lambda \in [0, 1]$. To see why this is the case define

$$
h(\lambda) \equiv B(\lambda)^2 - \frac{4(1-\alpha)(1-\rho)}{\alpha^2}
$$

and note that $h'(\lambda) = 2\rho B(\lambda) > 0$ and $h''(\lambda) = 2\rho^2 > 0$ for all $\lambda \in [0, 1]$. Hence the function is strictly convex, increasing and does not attain a minimum in the interval $[0, 1]$ since $h'(\lambda) > 0$ because $B(\lambda) > 0$ for all $\lambda \in [0, 1]$.

Second we show $0 < \Gamma'(\lambda) < 1$ for all $\lambda \in [0, 1]$. First let us show that $\Gamma(\lambda) > 0$ for all $\lambda \in [0, 1]$. From (5) it is sufficient to show that $B(\lambda) - \sqrt{B(\lambda)^2 - \frac{4(1-\alpha)(1-\rho)}{\alpha^2}}$ is positive for all $\lambda \in [0, 1]$. Note

$$
B(\lambda) > \sqrt{B(\lambda)^2 - \frac{4(1-\alpha)(1-\rho)}{\alpha^2}}
$$

$$
B(\lambda)^2 > B(\lambda)^2 - \frac{4(1-\alpha)(1-\rho)}{\alpha^2}
$$

$$
\frac{4(1-\alpha)(1-\rho)}{\alpha^2} > 0.
$$
Finally to show that $\Gamma(\lambda) < 1$ for all $\lambda \in [0, 1]$ it is sufficient to show $\Gamma(0) < 1$ since we have shown $\Gamma'(\lambda) < 0$ for all $\lambda \in [0, 1]$. Notice that for the negative root

$$\Gamma(0) = \frac{\alpha}{2} \left[ B(0) - \sqrt{B(0)^2 - \frac{4(1 - \alpha)(1 - \rho)}{\alpha^2}} \right] < 1$$

if the following holds

$$B(0) < \frac{2}{\alpha} + \sqrt{B(0)^2 - \frac{4(1 - \alpha)(1 - \rho)}{\alpha^2}}.$$  

We know that $\sqrt{B(0)^2 - \frac{4(1 - \alpha)(1 - \rho)}{\alpha^2}} > 0$ is adding to $\frac{2}{\alpha}$, then we just need to show that $B(0) < \frac{2}{\alpha}$ which comes down to showing that

$$1 - \rho + \left( \frac{1 - \alpha}{\alpha} \right)(2 - \rho) < \frac{2}{\alpha}$$

which is satisfied since this yields $(1 - \alpha)(2 - \rho) + \alpha(1 - \rho) < 2$ or $-\alpha - \rho < 0$. 

$\blacksquare$
References


